A MAGNETOAEROELASTIC ENERGY CONVERSION SCHEME

by

Jerome V. Burchett, Lieutenant, USN

BEE Rensselaer Polytechnic Institute 1959

Submitted in Partial Fulfillment of the

Requirements for the Degrees of

Master of Science in Electrical Engineering

and the

Professional Degree, Naval Engineer

at the

Massachusetts Institute of Technology

(May 1966)

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Signature of Author.....
Department of Naval Architecture

Certified by.....
Thesis Supervisor

Accepted by.....
Chairman, Departmental Committee on Graduate Students

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Jerome Vincent Burchett

Submitted to the Department of Naval Architecture and Marine Engineering on May 20, 1966, in partial fulfillment of the requirements for the Master of Science Degree in Electrical Engineering and the Professional Degree, Naval Engineer.

ABSTRACT

This thesis is a theoretical and experimental study of a scheme for converting energy from a streaming fluid through an elastic plate to an electric circuit. The model is a quasi-one-dimensional motion of the elastic plate under the influence of the supersonic fluid and a transverse magnetic field.

An analysis of the model leads to the equations of motion of the plate in the unrestrained case. By assuming traveling wave solutions, the dispersion relation, resulting from the coupled equations of motion, is solved. The boundary conditions are discussed and the resulting eigenvalue problem is stated. By the method of coupling modes, the effect of the coupling on the wave numbers is determined.

In the experiment, an elastic media is tested for response with electromagnetic and mechanical excitation. The supersonic flow is attained with a set of convergent-divergent nozzles feeding a test section. The combined plate and flow experiment is run and results discussed.

The conclusions are: (1) the theory predicts an instability in the natural modes; (2) the plate can be excited in the second modes and the frequency response increases with tension. (3) The effects of the instability in the combined experiment are masked by forced excitation probably due to the turbulent boundary layer.

Thesis Supervisor: James R. Melcher
Title: Associate Professor of Electrical Engineering

A MAGNETOHYDRODYNAMIC PUMP

by

James Eugene Hoover

Submitted to the Department of Naval Architecture and Marine Engineering on May 20, 1960, in partial fulfillment of the requirements for the Degree of Doctor of Science in Electrical Engineering and the Professional Degree, Naval Engineer.

ABSTRACT

This thesis is a theoretical and experimental study of a scheme for converting energy from a stationary fluid stream to electric power in an electric circuit. The model is a quasi-one-dimensional section of the electric plasma inside the tailpipe of the supersonic fluid and a transverse magnetic field.

Properties of two model loads in the equations of motion of the plasma in the working gas. By imposing irreversible boundary conditions, the dissipation ordinarily found the coupled equations of motion, is solved. The boundary conditions are discussed and the conflicting requirements is removed. By the method of coupling mode, the effect of the coupling on the load impedance is determined.

In the experiment, an electric media is tested for thermal with electrodynamic and thermodynamic initiation. The apparatus flow is achieved with a set of convergent-divergent nozzles creating a free section. The combined plate and flow measurements is run and results discussed.

The conclusions are: (1) the theory predicts an instability in the vertical modes. (2) the plasma can be ionized in the sonic modes and the Prandtl-Meyer expansion increases with Ionization. (3) The effects of the finiteness in the conductor experiment are caused by internal excitation probably due to the electrode boundary layer.

Thesis Supervisor: James L. Kerrebrock
Title: Associate Professor of Electrical Engineering

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CHAPTER I

Introduction

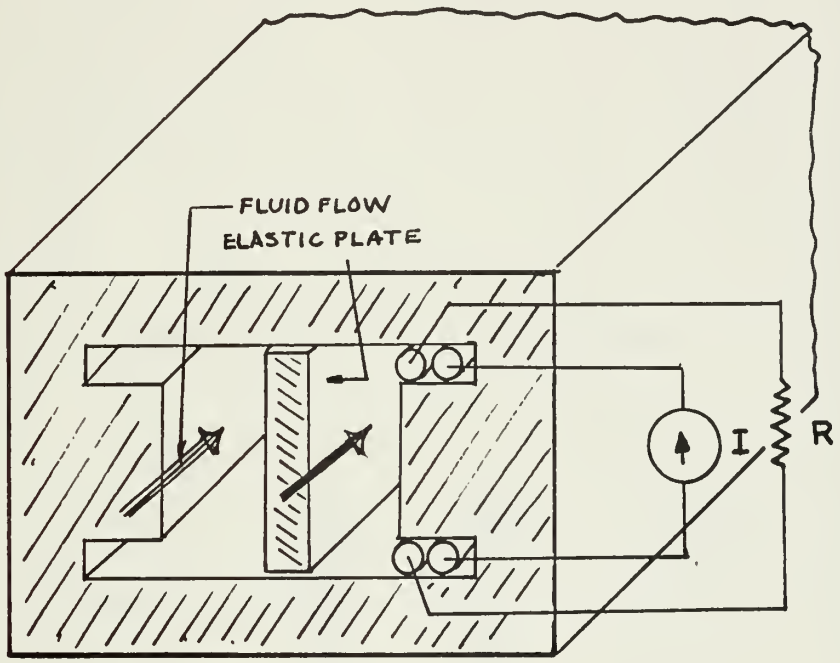
Energy conversion has historically been accomplished by rotating machinery, making use of small gap lengths and conduction in good conductors. Recent investigations have lead to novel static and dynamic converters. The feasibility of generating d.c. power from the interaction of a moving conducting fluid and an electromagnetic system has already been demonstrated.[1] Several schemes for A.C. power generation have been presented, however, these are restricted by the requirement of large magnetic fields and a good conducting fluid. An alternative method for converting energy is shown in Figure 1.1. It is a device for producing A.C. power by abstracting energy from the fluid flow.

The device consists of an elastic plate in a channel with a fluid flow on both sides. The plate is constructed of highly permeable material and is free to move in the transverse direction. The channel, also of highly permeable material, contains the fluid and copper conductors. One set of conductors is excited by a constant current (D.C.) source, causing a transverse magnetic field. The interaction between the fluid and the plate causes transverse motion of the plate, similar to the "flutter" motion in aero-elasticity. This motion provides a variable coupling of the first circuit with the second magnetic circuit, producing A.C. power, as indicated in the lumped parameter model shown in Figure 1.2.

Analytically the problem of the thin airfoil performing small lateral oscillations in a uniform stream of incompressible fluid

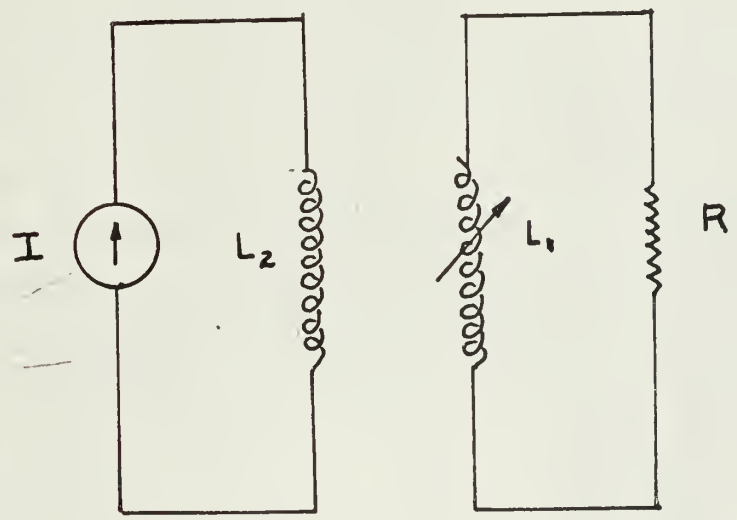
CHAPTER I

Introduction



A MAGNETOAEROELASTIC ENERGY CONVERSION SCHEME

FIGURE 1.1



LUMPED PARAMETER MODEL

FIGURE 1.2

was for many years the heart of all flutter prediction. The complete solution was first published in the United States by Theodorsen[2]. This approach has been further developed by many workers and is presented most comprehensively in the text, Aeroelasticity, by Bisplinghoff, etc.[3]

The definition of panel flutter is the self excited oscillation of the external skin of a flight vehicle when exposed to an airflow on one side The analysis of panel flutter has been pursued in great detail recently, but there is still some disagreement between theory and experiment. It appears that the theoretical considerations to date do not approach the situation described here. The tensions on the panel are significantly less than those to be applied in this device. The mechanism for flutter is not explained physically in the context of a feedback situation. However, the effects of aspect ratio, elastic foundations, damping and boundary support have been covered in detail. In the appendix a comparision of the prediction of flutter theory and the predictions of this theory is shown. The aeroelastician in general has confined his analysis to that of "infinite" flow, whereas in this scheme the confining effect of the rigid walls will be important. In dense fluids, there has also been investigation of flow induced vibrations of fla
follow
recent
elastic.[5,6] The latter is concerned with a statistical representation of the turbulent boundary layer as a forcing function. Here again, the models have infinite geometry and, therefore, are not directly applicable. Lastly there are several "handbook" solutions available for determining natural modes of vibration, for

example, see Den Hertog [7].

Melcher[8] has studied field coupled surface waves for both electrohydrodynamic and magnetohydrodynamic systems. Ketterer,[9] studying under him, investigated the interactions between a fluid stream and a fixed structure capable of supporting traveling waves By theory and experiment, it was shown that energy could be extracted from the flow in the form of oscillations.

In this study there is contact between the fluid and the elastic structure which was not present in the experiment conducted by Ketterer. The effect of this contact, a boundary layer which could be turbulent, may be considerable in comparison to the effect of an instability in the natural modes.

There are several methods of exciting the plate available. First, it could be located transversely in the flow path, similar to holding a piece of grass between the thumbs and blowing. However, the length of interaction would be limited and therefore, the amount of energy taken from the flow would be limited. Secondly, the plate could be fixed at one end only, and the excitation would be dependent on the trailing vorticies. This would be similar to the study reported by Eagleson.[4] Another method that has been suggested is to place the plate in a tube closed at the downstream end and put the combination in the supersonic flow. This would provide greater excitation since pressure variations considerably above stagnation pressure have been observed in such a configuration. The overall efficiency of this scheme is questionable, however.

The method chosen was considered both theoretically and experi-

mentally the simplest to study, with the added feature that the significant interaction length might produce a reasonably efficient device. In Chapter II, the device is studied by assuming a quasi-one dimensional motion of the plate. The resulting dispersion relationship is solved in Chapter III and the implications discussed. In chapter IV the longitudinal boundary conditions are treated. The experiment and results are described in Chapter V.

CHAPTER II

ANALYSIS OF DEVICE

The device will be modeled by assuming a quasi-one dimensional motion of the plate. The model, as seen in cross-section from a top view, is shown in Figure 2.1. The flow will be assumed inviscid and incompressible in this analysis since the objective is a device that can be tested in a wind-tunnel. The proposed orientation will also allow neglecting the effects of gravity. The fluid flow, the plate and the magnetic effects will be treated separately, and then the results will be combined to obtain the equations of motion

2.1 The Fluid Equations:

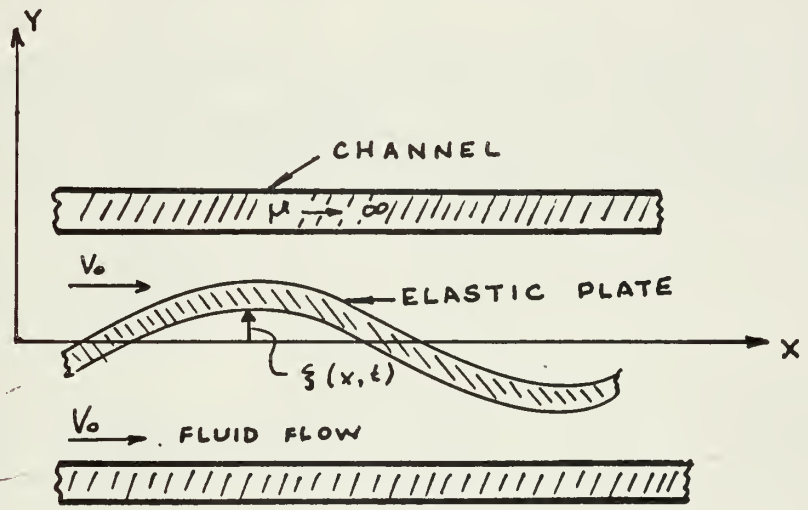
In the bulk of the fluid, if body forces are neglected, the momentum equation is

$$\rho \frac{Du_i}{Dt} + \frac{\partial p}{\partial x_i} = 0 \qquad (2.1)$$

for an inviscid fluid and the equation for continuity of mass is

$$\frac{D\rho}{Dt} + \rho \nabla \cdot u_i = 0 \qquad (2.2)$$

for compressible flow. We will assume small perturbations and that the waves are long compared to the significant dimensions, that is:



ANALYTICAL MODEL

FIGURE 2.1

$$v_x = v'_x(x,t) - V_o$$

$$v'_y = v_y(x,t)$$

$$p(x,t) = p'(x,t) - p_o << p_o$$

$$\rho(x,t) = \rho'(x,t) - \rho_o << \rho_o$$

where the primed quantities are the actual values and the unprimed quantities the perturbations. Inserting these into equations (2.1) and (2.2) and retaining only first order terms:

$$\rho_o\left(\frac{\partial v_x}{\partial t} + v_o \frac{\partial v_x}{\partial x}\right) + \frac{\partial p}{\partial x} = 0 \qquad (2.3)$$

$$\rho_o\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x}\right)\rho = 0 \qquad (2.4)$$

Initially, the thermodynamic process will be considered adiabatic, which allows relating the changes in pressure and density by a constant.

$$\partial\rho = c^2 \partial p$$

2.2 Magnetic Equations

A section of the magnetic circuit is shown in Figure 2.2. We assume a perfectly magnetic material so that the coil will induce a magnetic field in the circuit, with the H field existing

$$v''_\beta = v'_\beta(x,t) - \ldots$$

$$v'_\beta = v_\beta(x,t)$$

$$\rho(x,t) = \eta'(x,t) - \ldots < < \ldots$$

$$\rho(x,t) = \xi'(x,t) - \ldots < < \ldots$$

where the primed quantities are the actual values and the unprimed quantities the perturbations. Omitting those second (2.1) and subtracting only first order terms:

$$\rho_0 \left(\frac{\partial v_\beta}{\partial t} + v_\alpha \frac{\partial v_\beta}{\partial x} \right) = \frac{\partial p}{\partial x} = 0 \qquad (2.8)$$

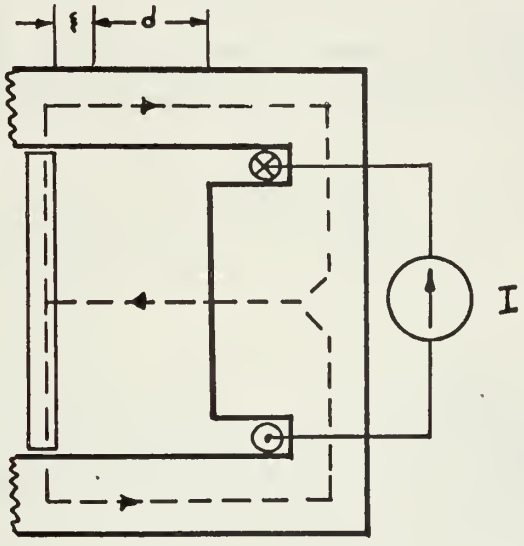
$$\rho_0 \left(\frac{\partial v_\beta}{\partial x} + \frac{\partial v_\beta}{\partial y} \right) = \left(\frac{\partial}{\partial x} + v_\alpha \frac{\partial}{\partial x} \right) = 0 \qquad (2.8)$$

Initially, the thermodynamic process will be considered adiabatic, which allows relating the changes in pressure and density by a constant.

$$p_0 = c \rho_0$$

2.2 Magnetic Equations

A section of the magnetic circuit is shown in Figure 2.2. We assume a perfectly magnetic material so that the coil will induce a magnetic field in the circuit, with the B field relating



MAGNETIC CIRCUIT

FIGURE 2.2

in the gap only. From Ampere's circuital law we get:

$$\oint H \cdot d\ell = \int_{gap} H \cdot d\ell = NI$$

$$\text{or} \quad H = \frac{NI}{d+\xi} = \frac{NI}{d}\left(\frac{1}{1+\xi/d}\right)$$

Define: $H_o = \frac{NI}{d}$ as the steady state H field. Therefore, the magnetic fields with reference to figure (2.1) will be:

$$H_y^{\ell} = H_o\left(\frac{1}{1-\xi/d}\right)$$

$$H_y^{u} = H_o\left(\frac{1}{1+\xi/d}\right)$$

where the superscripts refer to the respective sides of the plate. There are several methods available for computing the force on the plate. The most direct method is to use the Maxwell Stress Tensor described in detail in reference.[8] The force density in the y direction acting on the plate is the sum of the stresses acting in that direction on each of the faces of the plate. The stress tensor:

$$\tau_{ij} = \mu H_i H_j - \frac{\mu}{2}\delta_{ij} H_{kk}$$

In our case, where H_y is the only significant component, this becomes

$$\tau_{yi} = \frac{\mu}{2} H_y^2$$

in the eqs only. From Maxwell's external law we get:

$$\left\{ \begin{array}{l} \\ \end{array} \right. = \int_{\text{area}} \quad \text{at} -$$

or $\quad 0 = \frac{\partial \Gamma}{\partial t} - \frac{\Gamma}{2} \left(\frac{1}{\text{field}} \right)$

Define: $\Gamma^{\circ} = \frac{\partial \Gamma}{\partial t}$ as the steady state Γ field. Therefore, the magnetic field's auto reference to figure (2.1) will be:

$$\Gamma^{\chi}{}_{\lambda} = \Gamma^{\circ} \left(\frac{\quad}{\text{field}} \right)$$

where the asymmetrics refer to the respective sides of the plate. There are however methods available for computing the terms on the plate. The most direct method is to use the Maxwell Stress Tensor described in detail in reference.[3] The force density in the y direction acting on the plate is the sum of the stresses acting in that direction on each of the faces of the plate. The stress tensor:

$$\tau^{ij} = \mu \Gamma^{i} \Gamma^{j} - \frac{1}{2} \delta^{ij} \Gamma^{k} \Gamma^{k}$$

In our case, where Γ° is the only significant component this becomes

$$\tau^{yi} = \frac{1}{2} \mu \Gamma_{z}^{2}$$

and the force per unit area is

$$f_y = \frac{\mu}{2} H_o^2 \left(\frac{1}{(1+\xi/d)^2} - \frac{1}{(1-\xi/d)^2} \right)$$

Keeping only the linear terms, the force density is simply:

$$f_y = 2\mu_o H_o^2 \frac{\xi}{d}$$

Therefore, the force is proportional to the displacement and will have a destabilizing effect on the plate.

2 3 The Plate Equation

The plate has a density ρ and a modulus of elasticity E. Using simple beam theory, the bending moment at any point is proportional to the rate of change of the slope.

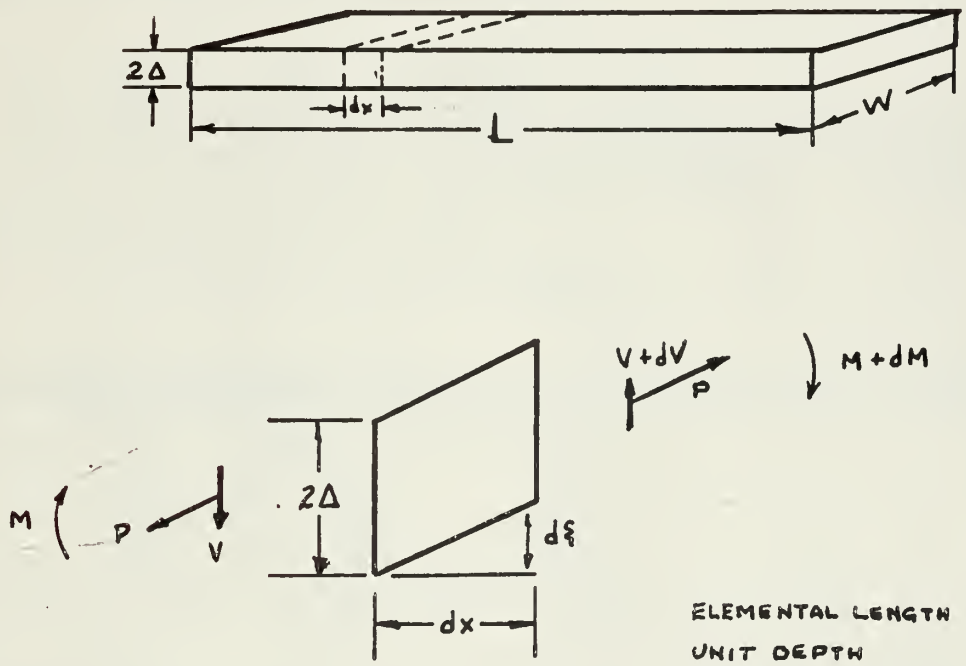
$$m = \frac{EI}{w} \frac{\partial^2 \xi}{\partial x^2} \quad \text{and} \quad dm = \frac{EI}{w} \frac{\partial^3 \xi}{\partial x^3} \, dx$$

Taking an elemental section as in Figure 2.3 we can indicate all of the forces and moments acting on it.

f = external force density = $2\mu_o H_o^2 \frac{\xi}{d} + (p^u - p')$

v = shear force per unit width.

p = tensile force per unit width.



ELASTIC PLATE AND ELEMENT

FIGURE 2.3

m = bending moment.

Now, summing the moments about the lower left hand corner:

$$-dm - vdx + pd\xi = 0 \qquad \text{or}$$

$$v = -\frac{dm}{dx} + p\frac{d\xi}{dx} = -\frac{EI}{w}\frac{d^3\xi}{dx^3} + p\frac{d\xi}{dx}$$

and the forces acting on the element.

$$f + \frac{dv}{dx} = \rho m 2\Delta \frac{\partial^2 \xi}{\partial t^2} \qquad \text{or}$$

$$f + p\frac{d^2\xi}{dx^2} - \frac{EI}{w} d\frac{d^4\xi}{dx^4} = \rho m 2\Delta \frac{\partial^2 \xi}{\partial t^2} \tag{2.5}$$

The three previous developments can be combined to give the equations of motion.

2.4 The Equations of Motion

Equation (2.4) can be rearranged and integrated directly.

$$-\frac{\partial v_y}{\partial y} = \frac{1}{\rho_o c^2}\left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x}\right) p + \frac{\partial v_x}{\partial x}$$

m = bending moment.

Now, summing the moments about the lower left hand corner:

$$-dm - vdx + pdl = 0 \qquad \text{or}$$

$$v = -\frac{dm}{dx} + p\frac{dz}{dx} = -\frac{EI}{w}\frac{d^3\zeta}{dx^3} + p\frac{dz}{dx}$$

and the forces acting on the element:

$$t = \frac{dv}{dx} = \rho wA\frac{d^2\zeta}{dt^2} \qquad \text{or}$$

$$t + p\frac{d^2z}{dx^2} - \frac{EI}{w}\frac{d^4\zeta}{dx^4} = \rho wA\frac{d^2\zeta}{dt^2} \qquad (2.3)$$

The three previous developments can be combined to give the equations of motion.

2.4 The Equations of Motion

Equation (2.4) can be rearranged and integrated directly.

$$-\frac{\partial\lambda}{\partial x} = \frac{1}{c_0^2}\left(\frac{\partial\varepsilon}{\partial t} + v_0\frac{\partial v_0}{\partial x}\right) + \frac{\partial v_0}{\partial x}$$

$$- v_y = \left(\frac{1}{\rho_o c^2} \left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x} \right) p + \frac{\partial v_x}{\partial x} \right) (y) \Big|_1^2 \qquad (2.6)$$

The transverse fluid velocity must be zero at the channel wall in both the upper and lower regions:

$$- v_y^{u'} = \left(\frac{1}{\rho_o c^2} \left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x} \right) p^{u'} + \frac{\partial v_x}{\partial x}^{u'} \right) (y \pm a) \qquad (2.7)$$

If one assumes no cavitation the transverse fluid velocity must equal the plate velocity at the plate.

$$-v_y^{u}(y= + \Delta + \xi) = \left(\frac{1}{\rho_o c^2} \left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x} \right) p^{u} + \frac{\partial v_x}{\partial x}^{u} \right) (\Delta+\xi-a)$$

$$-v_y^{\ell}(y = - \Delta+\xi) = \left(\frac{1}{\rho_o c^2} \left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x} \right) p^{\ell} + \frac{\partial v_x}{\partial x}^{\ell} \right) (-\Delta+\xi\xi a)$$

Since the perturbation ξ is small, $(\Delta+\xi -a) = -d$ and $v_y^{u'} (y = \pm\Delta+\xi)$

$= \frac{\partial \xi}{\partial t} + v_o \frac{\partial \xi}{\partial x}$,

the above two equations maybe added:

$$2\left(\frac{\partial \xi}{\partial t} + v_o \frac{\partial \xi}{\partial x}\right) = \left(\frac{1}{\rho_o c^2}\left(\frac{\partial}{\partial x} + v_o \frac{\partial}{\partial x}\right)(v_x^u - v_x^\ell + \frac{\partial}{\partial x}(p^u - p^\ell)\right) d \tag{2.8}$$

Equation (2.7), the plate equation, maybe rearranged and inserted in (2.3) and (2.8) to eliminate $(p^u - p^\ell)$, the pressure difference, from our equations:

$$\frac{\partial}{\partial x}\left(2\mu H_o^2 \frac{\xi}{d} - \frac{EI}{w}\frac{\partial^4 \xi}{\partial x^4} + p\frac{\partial^2 \xi}{\partial x^2} - 2\rho m\Delta \frac{\partial^2 \xi}{\partial t^2}\right) + \rho_o\left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x}\right) v = 0 \tag{2.9}$$

$$\left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x}\right)\xi - \frac{d}{2\rho_o c^2}\left(\frac{\partial}{\partial t} + v_o \frac{\partial}{\partial x}\right)\left(2\mu_o H^2 \frac{\xi}{d} - \frac{EI}{w}\frac{\partial^4 \xi}{\partial x^4} + p\frac{\partial^2 \xi}{\partial x^2} - 2\rho m\Delta \frac{\partial^2 \xi}{\partial t^2}\right) - \frac{d}{2}\frac{\partial}{\partial x} v = 0 \tag{2.10}$$

where we have defined $v = (v_x^u - v_x^\ell)$.

[illegible]

It will be convenient to have the equations in nondimensional form before further manipulations:

Variables: $\eta = \xi/d$, $v = \frac{v}{v_o}$, $\bar{x} = x/d$, $\bar{k} = kd$, $\bar{\omega} = \omega \frac{d}{v_o}$, $\bar{t} = \frac{v_o}{d}\, t$

Parameters: $\mu = \frac{\rho_o d}{\rho m \Delta}$, density ratio,

$M_a = \frac{c}{v_o}$, Fluid "Mach" number

$M_m = \frac{(P/2\Delta\rho m)^{1/2}}{v_o}$, Material "Mach" number

$H = \frac{\mu_o H^2 d}{\rho m v_o^2 \Delta}$, Magnetic pressure

$D = \frac{EI}{w2\rho m v_o^2 d^2 \Delta}$, Rigidity

We have normalized the parameters to a dynamic pressure ($\frac{1}{2}\rho v_o^2$) and then multiplied by the density ratio. This will simplify the solutions as well as provide an indication of the strength of the various forces or pressures compared to the fluid dynamic pressure.

We now multiply equation (2.9) by $\frac{d\rho_o c^2}{\Delta \rho_m v_o^2}$ and equation (2.10)

by $\frac{d^2}{2\Delta\rho_m v_o^2}$:

$$\mu M_a^2\left(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{x}}\right)\eta-\left(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{x}}\right)\left(H-D\frac{\partial 4}{\partial\bar{x}4}+M_m^2\frac{\partial^2}{\partial\bar{x}^2}\right)\eta-\frac{\mu}{2}M_a^2\frac{\partial}{\partial\bar{x}}v=0 \tag{2.11}$$

$$\frac{\partial}{\partial\bar{x}}\left(H-D\frac{\partial 4}{\partial\bar{x}^4}+M_m^2\frac{\partial}{\partial\bar{x}^2}-\frac{\partial^2}{\partial\bar{t}^2}\right)\eta+\frac{\mu}{2}\left(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{x}}\right)v=0 \tag{2.12}$$

The above form the equations of motion of the system in nondimensional form.

To solve these equations as they stand for specified forcing function would be difficult indeed. This approach has been followed on simplified equations, where the forcing function was a statistical representation of the boundary layer excitation. The approach we will follow is to look for instabilities in the free modes, when the source of oscillations is not a lossy one as it is in a turbulent boundary layer.

CHAPTER III

THE DISPERSION EQUATION

The dispersion equation, or the relationship between $\bar{\omega}$ and $\bar{k}$, will give an indication of the stability of the plate in the free modes. We will assume a delta function in time and space as the excitation, so that the equation of motion will have no driving function. In order to determine whether the structure can support traveling waves, we also assume that the solutions will be of the form $\eta = \hat{\eta}\, e^{j(\omega t - kx)}$ and $v = \hat{v}\, e^{j(\omega t - kx)}$. Inserting these into the equations of motion we get:

$$(\bar{\omega}-\bar{k})\,(\mu M_a^2 - H + D\bar{k}^4 + M_m^2\bar{k}^2 - \bar{\omega}^2)\hat{\eta} + \frac{\mu}{2}\,(M_a^2\bar{k})\hat{v} = 0 \tag{3.1}$$

$$-\bar{k}\,(H - D\bar{k}^4 - M_m^2\bar{k} + \bar{\omega}^2)\hat{\eta} + \frac{\mu}{2}\,(\bar{\omega}-\bar{k})\hat{v} = 0 \tag{3.2}$$

Setting the determinant of the coefficients equal to zero will insure that the equations of motion are compatible:

$$-\frac{\mu}{2}\left\{(\bar{\omega}-\bar{k})^2(\mu M_a^2 - H + D\bar{k}^4 + M_m^2\bar{k}^2 - \bar{\omega}^2) + M_a^2\bar{k}^2(H - D\bar{k}^4 - M_m^2\bar{k}^2 + \bar{\omega})\right\} = 0 \tag{3.3}$$

CHAPTER IV

THE DISPERSION EQUATION

This is the desired dispersion relationship, and its solution, $\bar{\omega} = f(\bar{k})$, and the inverse, $\bar{k} = f(\bar{\omega})$, will provide insight into the stability of the infinite plate. Before proceeding with the solution, there are several limiting cases which should be investigated. These in effect will separate the two systems.

1) If we allow the tension, P, on the plate to become infinite, the plate becomes a rigid boundary.

Thus, if $M_m \longrightarrow \infty$, we get:

$$(\bar{\omega}-\bar{k})^2\bar{k}^2 + M_a^2\bar{k}^2\,(-\bar{k}^2) = 0$$

or, in dimensional form:

$$(\omega - V_o k)^2 = M_a^2 k^2 \tag{3.4}$$

which is the expected dispersion equation for convective acoustic waves

2) Similarly if we allow the density of the fluid to approach zero, we find the equation of the plate alone.

$$\bar{\omega}^2 = -H + D\bar{k}^{-4} + M_m^2\bar{k}^2 \tag{3.5}$$

It is comforting to note that the magnetic force will have a destabilizing effect, while the tension and rigidity tend to stabilize the solution.

3) Allowing the speed of sound to become infinite will restrict the solution to the subsonic region. In dimensional form:

$$2\mu H^2 \frac{\xi}{d} - \frac{EI}{w}\frac{\partial^4 \xi}{\partial x^4} + P\frac{\partial^2 \xi}{\partial x^2} - 2\rho m\Delta \frac{\partial^2 \xi}{\partial t^2} + 2\rho_o V_o^2 \frac{\xi}{d} = 0$$

This is the equation for the plate with a dynamic pressure term which can be derived from Bernoulli's equation in a linearized form (assuming $\frac{\partial}{\partial t} \rightarrow 0$)

$$P_o + \frac{1}{2}\rho_o V_o^2 = P + \frac{1}{2}\rho\left(\frac{V_o d}{d-\xi}\right)^2 = P_o + P^{\ell} + \frac{1}{2}\rho V_o, + \rho V_o^2, \frac{\xi}{d}$$

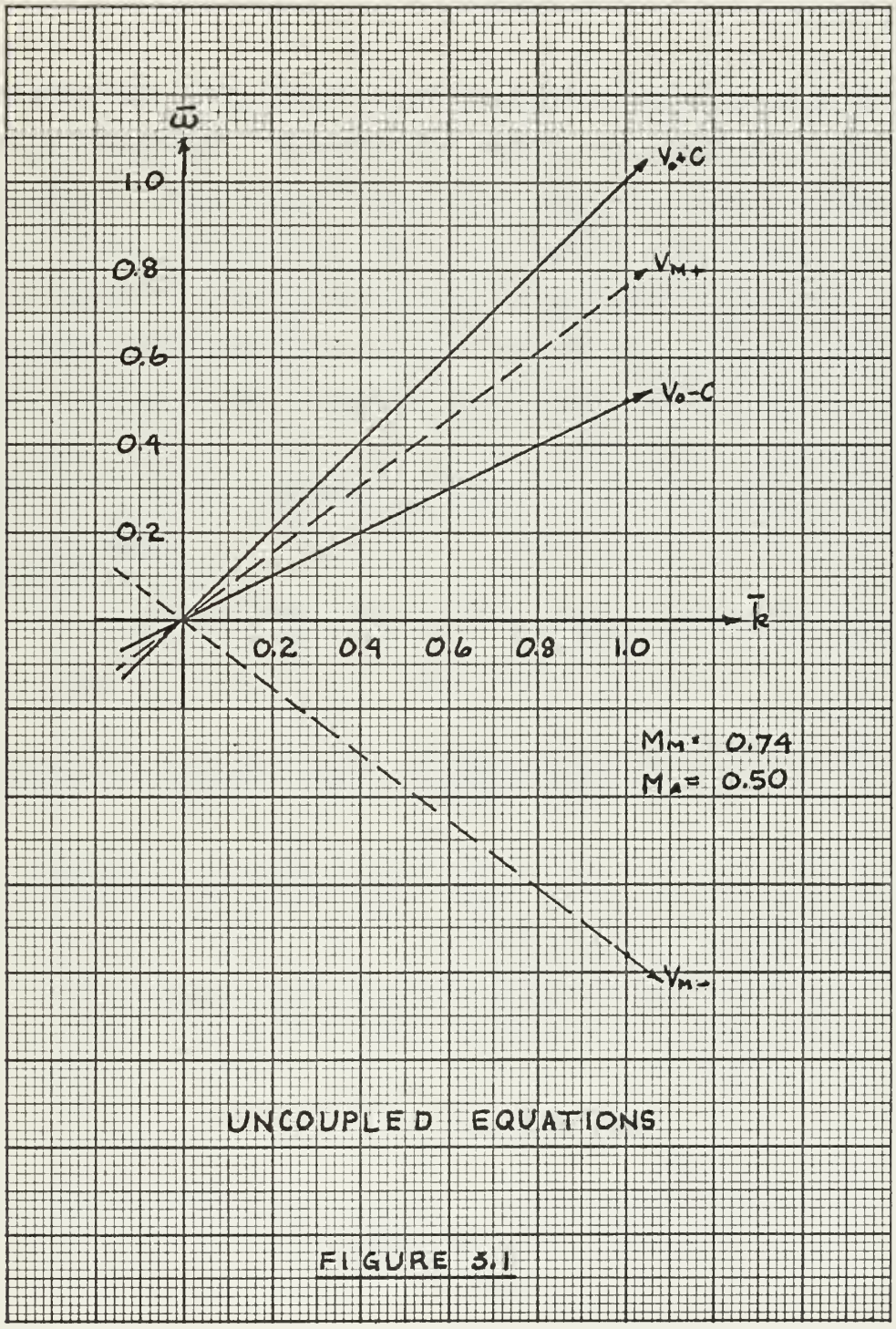
then:

$$P^u - P^{\ell} = 2\rho_o V_o^2 \frac{\xi}{d}$$

The above three checks indicate that the model and the analysis are consistant. It also appears that they are to some extent physical.

The third check gives a static instability which has been observed at subsonic speeds.[10]

The uncoupled equations (3.4) and (3.5) can be solved simply, since they are quadratic, and plotted, as in Figure 3.1. For the



UNCOUPLED EQUATIONS

FIGURE 3.1

rest of the analysis, some experimental considerations will govern the approach. Supersonic speeds are difficult to attain, and in general, the experimental apparatus is designed for a specified flow Therefore, we will take V_o to be a constant equal to twice the speed of sound. The rigidity of the plate is also a fixed quantity for a given experiment and the effect of its variation would be roughly equivalent to varying the tension on the plate. Since the plate tension is easily changed experimentally, it will be allowed to vary in the solution of the dispersion equation.

We now rearrange the dispersion equation as a polynomial in $\bar{\omega}$:

$$\bar{\omega}^4 + \bar{\omega}^3(-2\bar{k}) + \bar{\omega}^2\left(H - D\bar{k}^4 - M_m^2\bar{k}^2 - M_a^2 - \bar{k}^2M_a^2 + \bar{k}^2\right)$$

$$+ \bar{\omega}\,(2\bar{k})\left(-H + D\bar{k}^4 + M_m^2\bar{k}^2 + M_a^2\right)$$

$$+ (-\bar{k}^2)(M_a^2 - 1)(H - D\bar{k}^4 - M_m^2\bar{k}^2) \quad = 0 \tag{3.6}$$

and, using the computer, solve for real values of $\bar{k}$.

The results are shown in Figures 3.2, 3.3, and 3.4 for the following cases:

1) Figure 3.2 $M_m = .875$ (strong tension)

2) Figure 3.3 $M_m = .545$ (medium tension)

3) Figure 3.4 $M_m = .314$ (weak tension)

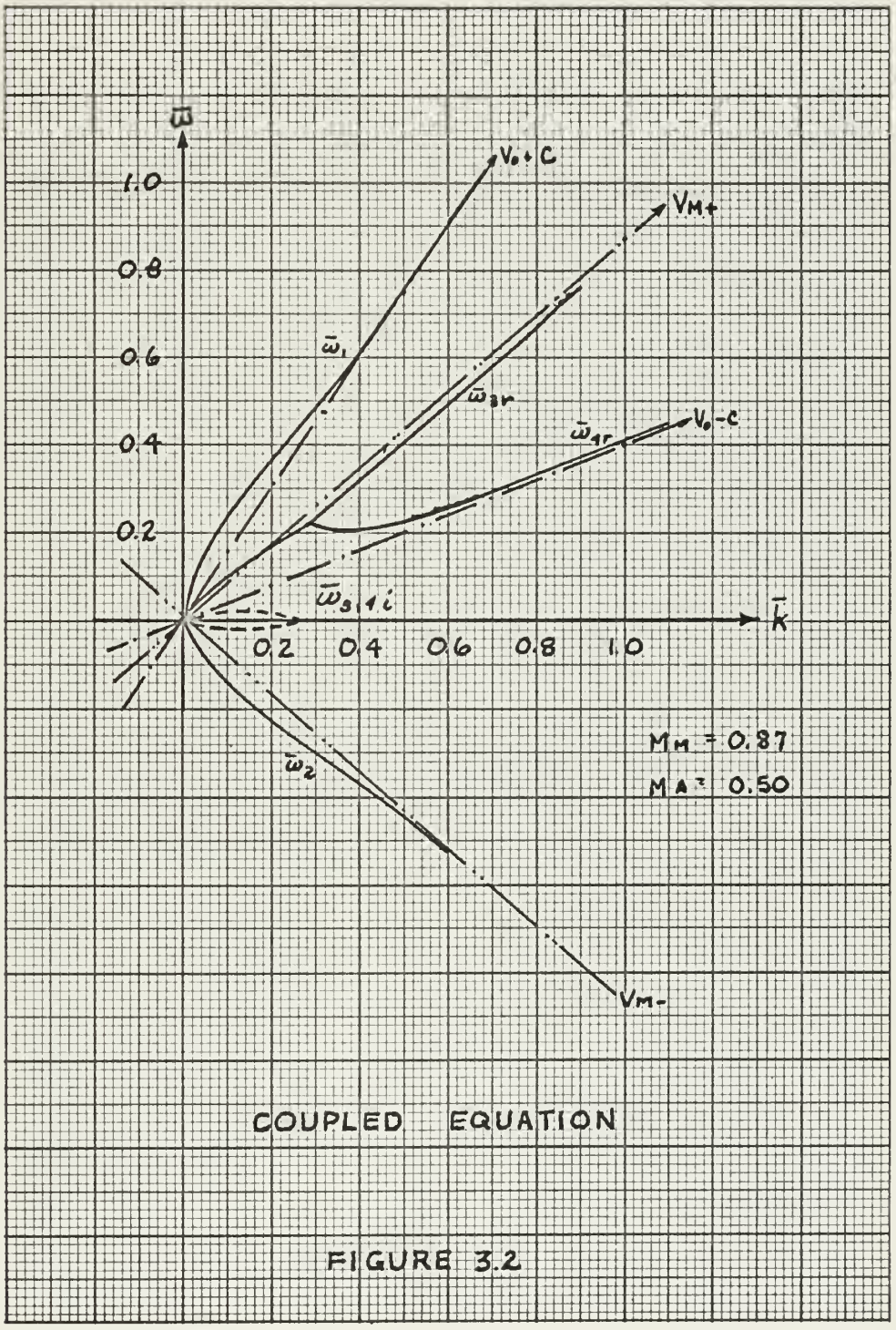
The solutions plotted in Figure 3.1 remain as the asympotes for these solutions. Returning to equations 3.3 and 3.4, we find in dimensional form:

$\omega = (V \pm c)k$ for the fluid

$\omega = \pm \left(\frac{p}{\rho}\right)^{1/2}$ for the plate

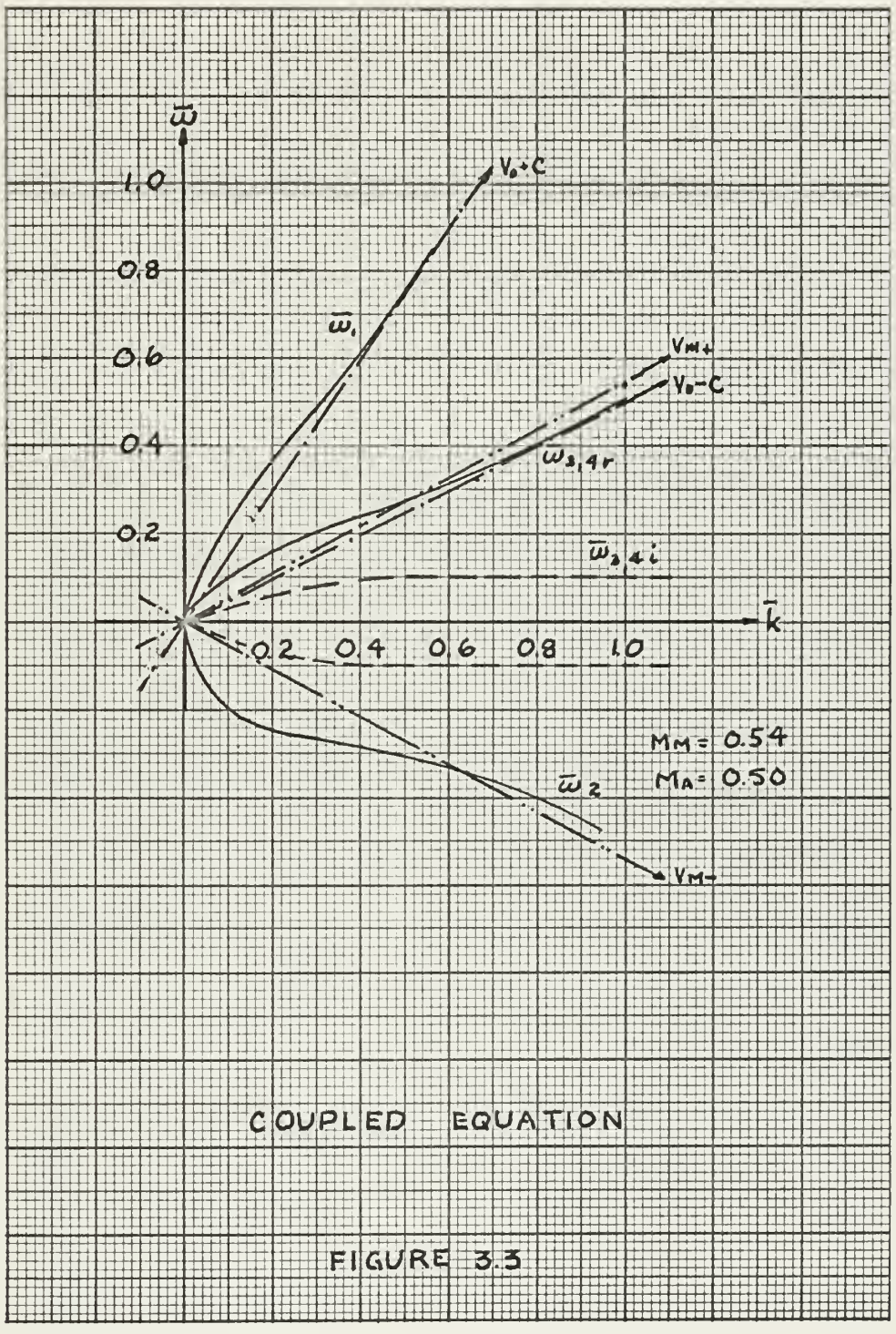
In the case of strong tension the coupling is weak. However, as the tension is decreased as in Figure 3.3 to a point where (v_{m+}), the wave speed in the plate, is approximately equal to (V-c), the "slow' fluid wave, the coupling becomes pronounced. Decreasing the tension further, decreases the coupling until the solutions are uncoupled

The two systems, the fluid and the plate, are in themselves stable at at least if the magnetic field is small. The above solutions indicate that when coupled together an unstable situa-



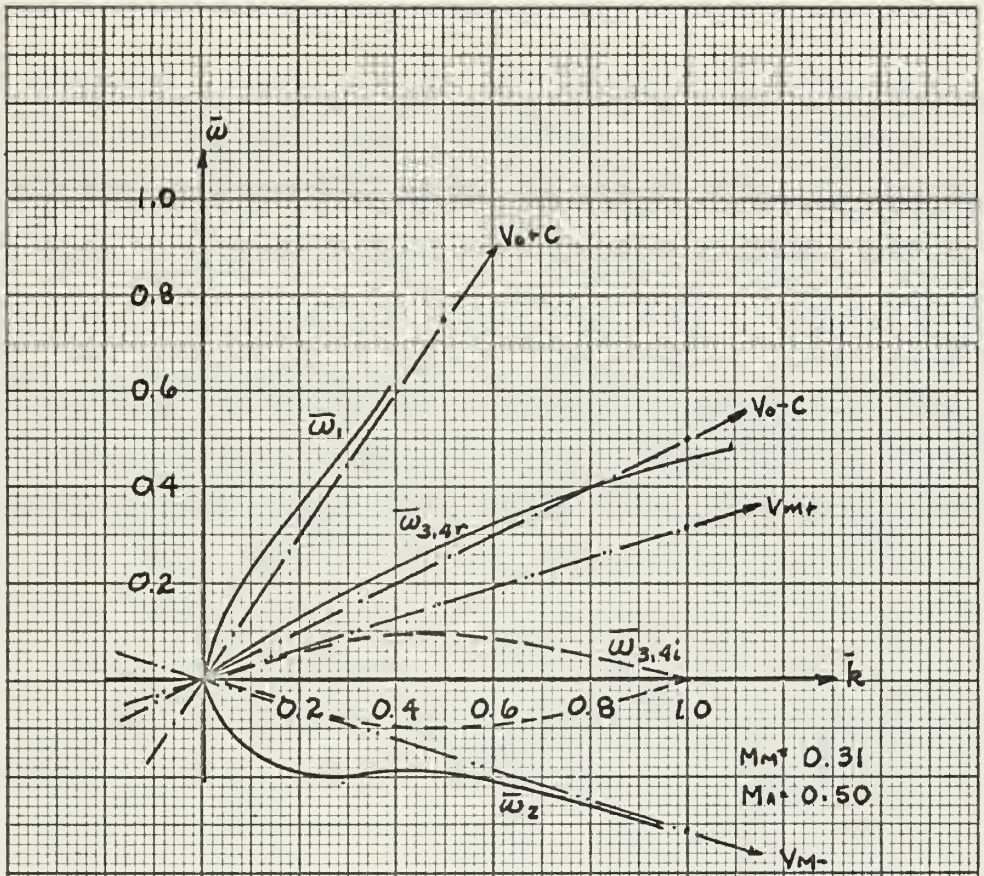
COUPLED EQUATION

FIGURE 3.2



COUPLED EQUATION

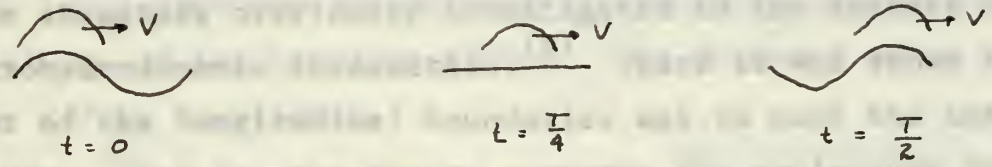
FIGURE 3.3



COUPLED EQUATION

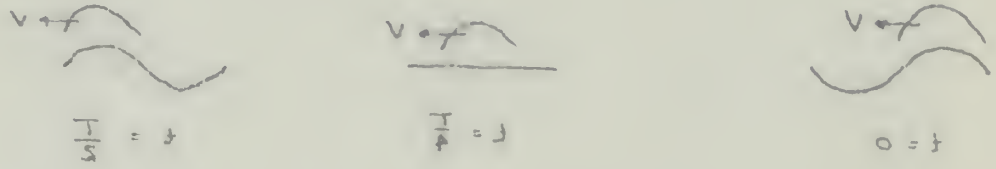
FIGURE 3.4

tion is produced. That is: $\bar{\omega}$ has a negative imaginary part and the assumed solution: $n=n\, e^{j(\omega t-kx)}$ has the form: $n = ne^{\alpha t+j(\omega t-kx)}$ such that the displacement grows in time. This is the result of a feedback mechanism within the coupled system. Let's assume a deflection of the plate in the second mode, with a positive amplitude over the leading half of the plate. This provides a compression wave in the fluid. As this disturbance is swept downstream, it arrives over the trailing half of the plate in time to reinforce the motion of plate. Then the motion is transmitted



back upstream through the plate, to arrive at the right time to start the procedure over again. This is the case when the velocities V_{m+} and V-c are equal. Since this is a distributed phenomenon over the half length of the plate, the mechanism will work for velocities that are approximately equal as well as exactly equal.

A complete discussion of the dispersion equation as it pertains to the infinite problem would not add significantly to the study of the device proposed. However, the steps will be outlined.


t = 0
t = T/4
t = T/2

Having solved for $\bar{\omega} = f(\bar{k}_{real})$, the complex values of $\bar{\omega}$ obtained are substituted and the equation solved for $k_r + jk_i = f(\omega_r + j\omega_i)$ to determine the type of instability involved. This can be done using the Bers-Briggs criteria,[11] with the help of computer technique giving a graphical display of both ω_r versus ω_i and k_r versus k_i as described by Mills.[12]

The dispersion equations shown in Figures 3.2 - 3.4 have characteristics which indicate that the basic theoretical model is physically meaningful. The coupling between the flow and the elastic media occurs at long wave lengths, with the short wavelengths stable and uncoupled. These are the properties of the stream structure previously investigated in the context of an electrohydrodynamic interaction.[8] There it was shown that the effect of the longitudinal boundaries was to make the instability absolute. That is, the theory supports the conjecture that growing oscillations can be produced.

CHAPTER IV

BOUNDARY CONDITIONS

The application of the boundary conditions will be approached by two methods, an exact method resulting in the eigenvalue problem and an approximate method called the coupling of modes approach. A simple linearized example will be covered first followed by a discussion of how to apply the boundary conditions. This leads to the statement of the eigenvalue problem. Next, the coupling of modes is discussed. This will give an indication of a more meaningful way of applying the boundary conditions, and an estimate of the coupling effect. The boundary value problem is not solved completely.

4.1 A Simple Example

Consider a membrane fixed at the ends. This is similar to the elastic plate if the rigidity and the fluid are neglected.

1 Equations of motion: $$\rho \frac{\partial^2 \xi}{\partial t^2} = p \frac{d^2 \xi}{dx^2}$$

2, Assume Solutions of the form: $\xi = (A \sin k_x + B \cos k_x)\, e^{j\omega t}$

3. Dispersion Relationship: $\omega^2 = p/\rho\, k^2$

CHAPTER IV

Boundary Conditions

The specification of the boundary conditions will be accomplished by the method of weighted residuals in the discretized version of the Galerkin and an approximate method called the coupling of modes approach. A simple illustrative example will be presented first followed by a discussion of how to apply the boundary conditions. This leads to the statement of the variational principle. Then, the coupling of modes is discussed. This will give an understanding of a more meaningful way of applying the boundary conditions, and an statement of the coupling method. The boundary value problem is not solved completely.

4.1 Simple Example

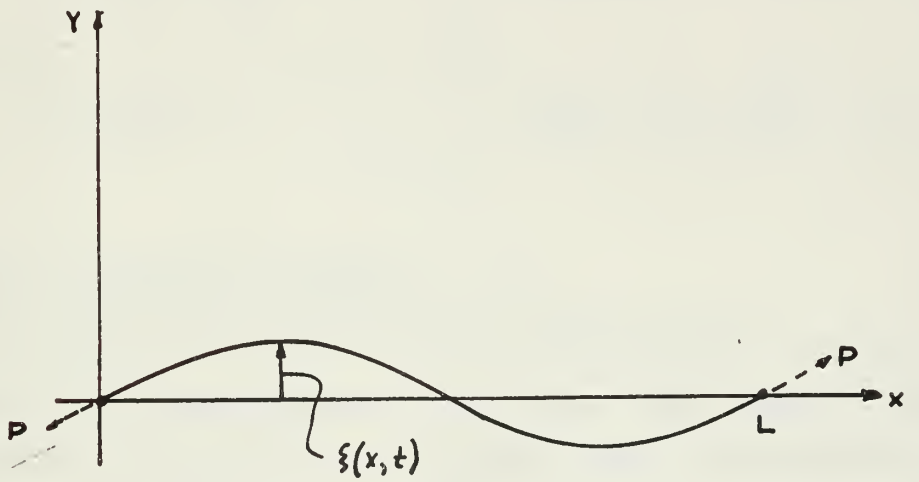
Consider a semi-infinite fluid at the ends. This is similar to the elastic plate if the elasticity and the fluid are neglected.

1. Equation of motion:

$$\frac{\partial^2 \phi}{\partial x^2} = 0 \frac{d^2 \phi}{dx^2}$$

2. Assume Solutions of the Form $\phi = (A \sin k^x + B \cos k^x) e^{j\omega t}$

3. Dispersion Relationship $\omega^2 = k^2 c^2$



TENSILE STRESS - P
DENSITY - ρ

A MEMBRANE FIXED AT THE ENDS

FIGURE 4.1

4. Boundary Conditions:

1) $\xi(0,t) = 0 \qquad B = 0$

2) $\xi(L,t) = 0 \qquad A \sin kL = 0 \quad \text{and} \quad k = \frac{n\pi}{L}$

5. Solution:

$$\xi(x,t) = \mathrm{Re} \left\{ \left(An \sin \frac{n\pi x}{L} \right) e^{j\left(\left(\frac{p}{\rho} \right)^{\frac{1}{2}} \frac{n\pi}{L} \right) t} \right\}$$

4.2. Method of Characteristics

In the example the basic steps to the problem solution are stated. There is a significant change when the fluid is added, especially when it is allowed to have a gross translational motion. The question arises - at what point may one apply boundary conditions on the flow such that they are meaningful? Were the fluid motionless, the boundary conditions must be applied at two points in space. This is also true if the velocity of the fluid is less than the speed of sound in the fluid. From the dispersion equation, however, the instabilities occur at supersonic speeds only.

In order to understand this more clearly, the method of characteristics is useful. This method uses the general solutions of the wave equation for the transient case:

$$f = f + (x-ct) + f - (x+ ct)$$

4. Boundary Conditions:

1) $f(0,t) = 0 \qquad t > 0$

2) $f(\xi,t) = \xi \qquad t$ and $\xi = \frac{x}{L}$

5. Solution:

$$f(\xi,t) = \xi \left\{ \left(\ln \frac{\alpha \tau_0}{L} \right) \alpha' \left(\frac{\tau_0^2}{\alpha} \right)^{\frac{1}{2}} \frac{L}{x} \right\} t$$

6.3. Method of Characteristics

In the example the basic steps to the problem solution are stated. There is a significant change when the fluid is added, especially when it is allowed to have a free boundary motion. The question arises - at what point may one apply boundary conditions on the flow such that they are meaningful? Were the fluid motionless, the boundary conditions must be applied at two points in space. This is also true if the velocity of the fluid is less than the speed of sound in the fluid. From the separation equation, however, the information exists at supersonic speeds only.

In order to understand this more clearly, the method of characteristics is useful. This method says the general solutions of the wave equation for the transient case.

$$f = F + G \qquad \text{or} \qquad f = f(x+at) + f(x-at)$$

where f+ is a wave propagating in the plus x direction and f- a wave in the negative x direction. The solutions are represented by the third dimension of a graph where the first two are time and space. Thus a disturbance can be followed in both time and space.

Let : $\alpha = x + ct$ and $\beta = x - ct$

Assume a disturbance: $p = P_o \quad -a < x < +a$

$= 0$ elsewhere.

for the fluid flow shown in Figure 4.2.

In Figure 4.3 the disturbance is shown.

Lines representing constant α and β can be drawn on the graph Those originating at $x = \pm a$ are the most interesting since they show the progress of the disturbance. For example, at $t = t_o$ the position of the disturbance can be determined from:

$f = f+(\beta) \quad t = t_o + f-(\alpha) \quad t = t_o$

The slope of the lines is of interest since, if the lines of β had a positive slope, the point $x = -L$, would never know the disturbance This is the case when the fluid is allowed to have motion at a velocity greater than the speed of sound. For a translating media, we must redefine:

$\alpha = x +(v+c)t \qquad$ and $\qquad \beta = x+(v-c)t$

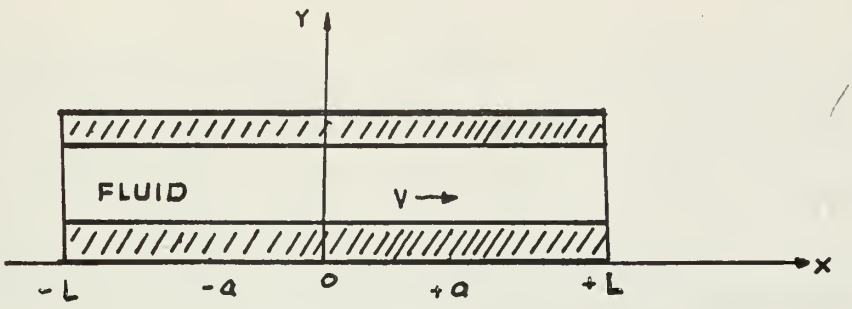
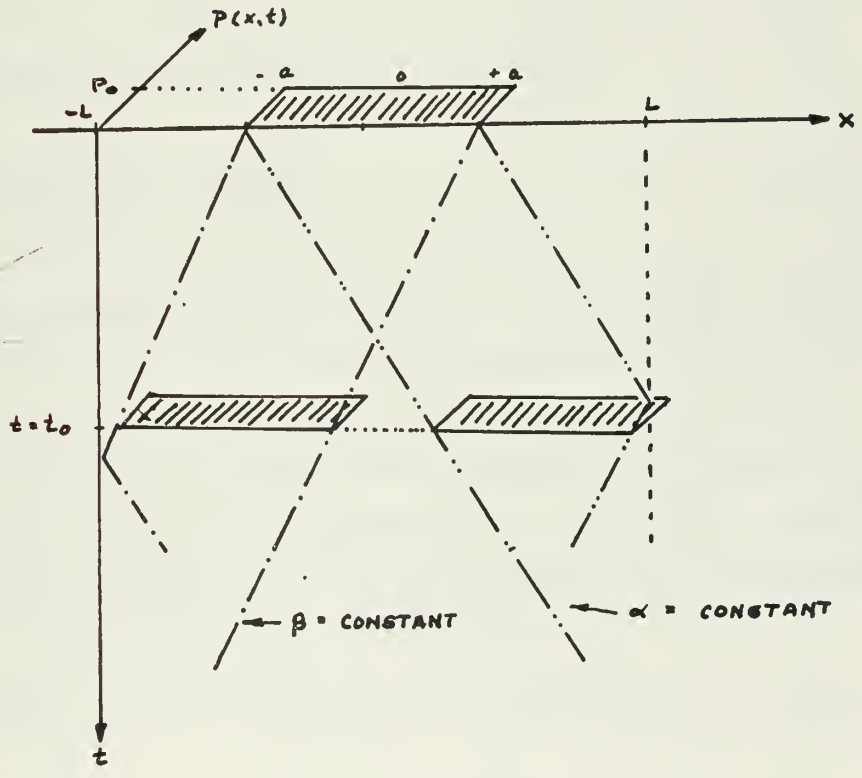


FIGURE 4.2



FLUID WAVE PROPAGATION

FIGURE 4.3

as the characteristic solutions to the convecting wave equation. Then (v-c), the slope of the β lines, becomes positive for $v > c$, and points upstream will not be effected by events downstream.

This implies, in the scheme proposed, that the fluid boundary conditions must be applied at the leading edge of the plate in order to determine their effect on the interaction. The fluid will not know what is happening at the trailing edge until it gets there. There are two variables on the flow, the pressure and velocity, and their values at the point $x = 0$, as a function of time, form the constraints on the flow.

4.3 The Eigenvalue Problem

The dispersion equations was sixth order in $\bar{\omega}$. The six boundary conditions are the two specified for the flow, plus the four on the plate. From simple beam theory the latter are determined from the type of support-free, pinned, or clamped. If we assume the ends are clamped, this implies that that the deflection and slope, $\frac{d\xi}{dx}$, are zero at $x = 0$ and $x = L$. Since $v(x,t)$ and $p(x,t)$ are defined as difference quantities (above and below the plate), their values are zero at $x = 0$.

Returning to the equations of motion this can be stated simply.

$$(\bar{\omega} - \bar{k})(\mu Ma\text{-P.E.})\hat{\eta} + \frac{\mu}{2}(Ma^2\bar{k}^2)\hat{v} = 0 \qquad (4.1)$$

$$\bar{k}(\text{P.E.})\hat{\eta} + \frac{\mu}{2}(\bar{\omega}-\bar{k})\hat{v} = 0 \qquad (4.2)$$

$$(P.E.)\eta + p = 0 \tag{4.3}$$

where $P.E. = H - D\bar{k}^{-4} - M_m^2\bar{k}^{-2} + \bar{\omega}^{-2}$

As in the example, assume a solution of the form:

$$\eta(x,t) = \mathrm{Re}\left\{ \sum A_n e^{-jk_n x} e^{j\omega_n t} \right\}$$

and rewrite equation (4.1) and (4.2).

$$p = -\frac{1}{P.E.}\,\eta \tag{4.4}$$

$$v = \frac{\mu}{2}\,\frac{(\bar{\omega}-\bar{k})}{\bar{k}(P.E.)}\,\eta = f_1\eta \tag{4.5}$$

applying the boundary conditions:

(1) $\eta(0,t) = 0 \qquad \sum A_n = 0$ (4.6a)

(2) $\frac{\partial \eta}{\partial x}(0,t) = 0 \qquad \sum jk_n A_n = 0$ (4.6b)

(3) $\eta(L,t) = 0 \qquad \sum A_n e^{jk_n L} = 0$ (4.6c)

(4) $\frac{\partial}{\partial x}\,\eta(L,t) = 0 \qquad \sum 2jk_n A_n e^{jk_n L} = 0$ (4.6d)

(5) $p(0,t) = 0 \qquad \sum 2\left(-\frac{1}{(P.E.(k_n)}\right) A_n = 0$ (4.6e)

$$(6)\quad v(0,t) = 0 \qquad \sum 2f_1(k_n)\, A_n = 0 \qquad (4.6f)$$

Rearranging in matrix form:

$$\Delta(\bar{\omega},\bar{k}) =$$

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
\bar{k}_1 & \bar{k}_2 & \bar{k}_3 & \bar{k}_4 & \bar{k}_5 & \bar{k}_6 \\
e^{j\bar{k}_1 L} & e^{j\bar{k}_2 L} & e^{j\bar{k}_3 L} & e^{j\bar{k}_4 L} & e^{j\bar{k}_5 L} & e^{j\bar{k}_6 L} \\
\bar{k}_1 e^{j\bar{k}_1 L} & \bar{k}_2 e^{j\bar{k}_2 L} & \bar{k}_3 e^{j\bar{k}_3 L} & \bar{k}_4 e^{j\bar{k}_4 L} & \bar{k}_5 e^{j\bar{k}_5 L} & \bar{k}_6 e^{j\bar{k}_6 L} \\
-\frac{1}{P.E.(\bar{k}_1)} & -\frac{1}{P.E.(\bar{k}_2)} & -\frac{1}{P.E.(\bar{k}_3)} & -\frac{1}{P.E.(\bar{k}_4)} & -\frac{1}{P.E.(\bar{k}_5)} & -\frac{1}{P.E.(\bar{k}_6)} \\
f_1(\bar{k}_1) & f_1(\bar{k}_2) & f_1(\bar{k}_3) & f_1(\bar{k}_4) & f_1(\bar{k}_5) & f_1(\bar{k}_6)
\end{bmatrix}$$

In theory then the equations $\Delta(\bar{\omega}_{jk}) = 0$ and $D(\bar{\omega},\bar{k}) = 0$ can be solved. Assuming a complex ω, the six k's are found from $D(\omega,k) = 0$. These values are substituted in to Δ. This procedure is continued until $\Delta = 0$; both it's real and imaginary parts, since it is complex In practice another method, that of weak coupling of modes will be used to attack the problem.

4.4 Method of Coupling of Modes

One might ask the question: "If two systems are individually stable, why would a weak coupling between the two result in totally different motions?" The answer is that the basic solutions for the two systems are approximately correct, and that the changes can be predicted within certain limitations. First, the systems must be stable by themselves. In this problem, this restriction will not allow including the effect of the magnetic stress on the plate since it has destabilizing influence. Secondly, the coupling between the two systems must be weak. Within these restrictions, mutual coupling of modes will allow simplifications based on a physical understanding of the system.

In theory, the approach is quite simple.[13] A complicated system is first divided into its component parts. The equations of motion for the isolated systems are solved exactly, and the solutions are expressed in the "normal modes" of the element. The complex system is then assumed to be made up of these elements weakly coupled. The motions within the system are described by the elemental motions which are slightly perturbed by the coupling.

As an example, in the system under consideration the elemental parts are the fluid and the plate. Assuming the fluid motionless and the plate of negligible rigidity (a membrane), each will propagate disturbances as waves, both a forward wave and a backward wave in each system. As seen from the solution

3.4 Method of Coupling of Modes

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As an example, in the system under consideration the elemental parts are the fluid and the plate. Assuming the fluid motionless and the plate of negligible rigidity (a membrane), each will propagate disturbances as waves, both a forward wave and a backward wave in each system. As seen from the solution

of the dispersion relationship, the coupling will occur between the forward membrane wave and the "backward" fluid wave. Therefore, the other two waves can be neglected. The motion of the system will be that of these two waves slightly altered by the coupling.

4.5 Elemental Equations in the Coupled Mode Form

1. Fluid Equations

The fluid equations can be greatly simplified to get an approximation to the solution.

$$\frac{\partial v(x,t)}{\partial x} = - \frac{1}{\rho_o (v \pm c)} \frac{\partial p(x,t)}{\partial t} \qquad (4,8a)$$

$$\frac{\partial p(x,t)}{\partial x} = -\rho_o \frac{\partial v(x,t)}{\partial t} \qquad (4.8b)$$

These coupled equations are most easily solved if a method of decoupling them can be found Following, Louisell,[13] the decoupled form is derived from linear combinations of the above equations. That is; assume a normal mode form:

$$b \pm (x,t) = \frac{1}{2} (\rho)^{\frac{1}{2}} \left(V(x,t) + \frac{1}{\rho_o (v \pm c)} p(x,t) \right)$$

where the mode amplitudes are normalized so that the square of the amplitudes represents energy carried by the mode.

Multiplying equation (4 8b) by the fluid impedance ($\frac{1}{\rho_o(v\pm c)}$) and adding and subtracting from equation (4.8a) will give the desired decoupled form:

$$\frac{\partial}{\partial x}\left(V \pm \frac{1}{\rho_o(V\pm c)} P\right) = - \frac{1}{V\pm c}\frac{\partial}{\partial t}\left(V + \frac{1}{\rho_o(V-c)} P\right)$$

or

$$\frac{\partial}{\partial x}\left(b \pm (x,t)\right) = - \frac{1}{V\pm c}\frac{\partial}{\partial t}\left(b \pm (x,t)\right)$$

Assume that the time dependence will be of the form $e^{j\omega t}$,

then: $b \pm (x,t) = a \pm (x)\, e^{j\omega t} + a^{*}\pm(x)\, e^{-j\omega t}$

Now if $v(x,t) = \mathrm{Re}\ \bar{V}(x)e^{j\omega t}$

then $a \pm (x) = \frac{1}{2}(\rho)^{\frac{1}{2}}\left(\bar{V}(x) + \frac{1}{\rho_o(v\pm c)}\bar{P}(x)\right)$

and equations (4.8a) and (4.8b) become

$$\left(\frac{\partial}{\partial x} + jk\right)a+(x) = 0 \qquad \left(\frac{\partial}{\partial x} - jk\right)a^{*}+(x) = 0 \qquad (4.9)a$$

$$\left(\frac{\partial}{\partial x} - jk\right)a-(x) = 0 \qquad \left(\frac{\partial}{\partial x} + j\bar{k}\right)a^{*}-(x) = 0 \qquad (4.9b)$$

where $k = \frac{\omega}{v \pm c}$

2 The Plate Equations

Again simplifying assumptions must be made. Neglecting the rigidity and the external forces on the plate, the normal mode form of the equations for this element can be derived in an analogous manner.

$$\frac{\partial v(x,t)}{\partial x} = \frac{1}{p}\frac{\partial t}{\partial t} = \frac{\partial S(x,t)}{\partial t} \qquad \text{where } v(x,t) = \frac{\partial \xi}{\partial t}$$

$$\frac{\partial v}{\partial t}(x,t) = \frac{p}{\rho}\frac{\partial S(x,t)}{\partial x} \qquad S(x,t) = \frac{\partial \xi}{\partial x}$$

then

$$b \pm (x,t) = \frac{1}{2}(\rho)^{\frac{1}{2}}\left(V(x,t) \pm \frac{\rho}{p} S(x,t)\right)$$

and

$$\frac{\partial}{\partial x}(b \pm (x,t)) = \mp \left(\frac{\rho}{p}\right)^{\frac{1}{2}} \frac{\partial}{\partial t}(b \pm (x,t))$$

therefore,

$$\left(\frac{\partial}{\partial x} - jk_2\right) a_2+(x) = 0 \qquad \left(\frac{\partial}{\partial x} + jk_2\right) a_2^* + = 0 \qquad (4.10a)$$

$$\left(\frac{\partial}{\partial x} + jk_2\right) a_2-(x) = 0 \qquad \left(\frac{\partial}{\partial x} - jk_2\right) a_2^* - = 0 \qquad (4.10b)$$

where $k = \frac{\omega}{c}$

5. The Plate Equations

Again dimensionless coordinates must be used. Developing the validity and the neutral curves in the plane, the normal mode form of the equations for this element can be derived in an analogous manner.

$$\frac{\partial^2 v(x,t)}{\partial x^2} + \int_0^1 \frac{\partial^2 v(\xi,t)}{\partial \xi^2} \, d\xi = \frac{\partial^2 v(x,t)}{\partial t^2} \quad \text{where } v(x,t) = \frac{v}{c}$$

$$\frac{\partial^2 v}{\partial x^2}(x,t) + \int_0^1 \frac{\partial^2 v}{\partial \xi^2}(\xi,t) \, d\xi = \frac{\partial^2 v}{\partial t^2}$$

then

$$B \equiv (x,t) = \frac{1}{2} \left(\int_0^1 (\xi)(x,t) \right) + \frac{\partial}{\partial x}(x,t)$$

and

$$\frac{\partial^2}{\partial x^2} (B \pm (x,t)) = \pm \left(\frac{\partial^2}{\partial t^2} \right)^{\frac{1}{2}} (x,t)$$

therefore,

$$\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial^2}{\partial t^2} \right)^{\frac{1}{2}} \right) \phi^+(x) = 0 \qquad \left(\frac{\partial^2}{\partial x^2} - \left(\frac{\partial^2}{\partial t^2} \right)^{\frac{1}{2}} \right) \phi^- = 0$$

(5.10a)

$$\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial^2}{\partial t^2} \right)^{\frac{1}{2}} \right) \phi^+(x) = 0 \qquad \left(\frac{\partial^2}{\partial x^2} - \left(\frac{\partial^2}{\partial t^2} \right)^{\frac{1}{2}} \right) \phi^- = 0$$

(5.10b)

where

$$k_2 = \frac{\omega}{(P/\rho)^{\frac{1}{2}}}$$

4:6 Coupling Between the Two Elements

In section B, we derived the normal mode form of the element equations In effect the single variables in equations (4.8a) and (4 8b) have been replaced by a combination of the two variables. To put the previous equations in coupled mode form, it is necessary to make linear combinations of the elemental forms. This is done in such a manner that when the coupling is reduced to zero, the equations reduce to those derived for each element separately.

$$\frac{\partial a_1}{\partial x} = c_{11}a_1 + c_{12}a_2 + c_{13}a_1^* + c_{14}a_2^* \qquad (4.11a)$$

$$\frac{\partial a_2}{\partial x} = c_{21}a_1 + c_{22}a_2 + c_{23}a_1^* + c_{24}a_2^* \qquad (4.11b)$$

$$\frac{\partial a_1^*}{\partial x} = c_{31}a_1 + c_{32}a_2 + c_{33}a_1^* + c_{34}a_2^* \qquad (4.11c)$$

$$\frac{\partial a_2^*}{\partial x} = c_{41}a_1 + c_{42}a_2 + c_{43}a_1^* + c_{44}a_2^* \qquad (4.11d)$$

where

$$a_1 = \frac{1}{2}\left((\rho f)^{1/2} v_f(x) + \frac{1}{\rho_o (v \pm c)} \bar{p}(x)\right)$$

where

$$k^2 = \frac{\omega}{(EI/\rho)^{1/4}}$$

5.6 Coupling Between the Two Planes

In section 3, we derived the normal mode form of the elastic equations. In either the simple variable in equations (3.3a) and (4.35) have been replaced by a combination of the two variables. To put the previous equations in coupled mode form, it is necessary to make linear combinations of the elementary forms. This is done in such a manner that the coupling is reduced to zero, the equations reduce to those derived for each element separately

$$\frac{\partial^4 \xi_1}{\partial x^4} = c_{11}\xi_1 + c_{12}\xi_2 + c_{13}\xi_3 + c_{14}\xi_4 \tag{6.11a}$$

$$\frac{\partial^4 \xi_2}{\partial x^4} = c_{21}\xi_1 + c_{22}\xi_2 + c_{23}\xi_3 + c_{24}\xi_4 \tag{6.11b}$$

$$\frac{\partial^4 \xi_3}{\partial x^4} = c_{31}\xi_1 + c_{32}\xi_2 + c_{33}\xi_3 + c_{34}\xi_4 \tag{6.11c}$$

$$\frac{\partial^4 \xi_4}{\partial x^4} = c_{41}\xi_1 + c_{42}\xi_2 + c_{43}\xi_3 + c_{44}\xi_4 \tag{6.11d}$$

where

$$c_{ij} = \frac{1}{I} \left(\phi_i(x) \frac{\partial^4 \phi_j(x)}{\partial x^4} + \frac{\rho' A'}{\rho A} \psi_i(x) \right.$$

$$a_2 = \frac{1}{2}(\rho_f)^{1/2}(\bar{V}_p(x) \mp \frac{\rho}{\rho_p} S(x))$$

and

$$c_{22} = -jk_2(1 + \ell/4\rho_f k_2^2)$$

$$c_{32} = -j\frac{I}{4\rho_f k_2}$$

$\ell \triangleq$ coupling coefficient

In order to make the problem more tractable we must make use of our previous results. The first is that coupling between the two systems will occur when the two wave members, k_1 and k_2, are approximately equal. Since $k_1 = \frac{\omega}{v-c}$ and $k_2 = \frac{\omega}{V_m}$ and in Chapter II we saw that the coupling occured when $V_{m+} = V-c$, we can therefore neglect the fast fluid wave (V+c) and the backward plate wave (V_{m-}).

Secondly, we must determine if, in fact, the coupling between the two systems is weak. Returning to the coupled dispersion relationship and inserting our previous assumption that the rigidity and the magnetic field are zero, we have:

$$-(\bar{\omega}-\bar{k})^2(\mu M_a^2 + M_m^2\bar{k}^2 - \bar{\omega}^2) + M_a^2\bar{k}^2(-M_m^2\bar{k}^2+\bar{\omega}^2) = 0$$

or in dimensional form and rearranged.

$$\left\{(\omega-V_o k)^2-c^2k^2\right\}\left\{V_m^2k^2-\omega^2\right\} = \frac{\mu c^2}{\partial^2}(\omega-V_o k)^2$$

where the term on the right represents the coupling between the two systems. Now let the perturbation of the elemental wave numbers be small:

$$k_1^* = \frac{\omega}{V-c} + \delta_1 \qquad \text{where } \delta_1 << \frac{\omega}{V-c}$$

$$k_2^* = \frac{\omega}{V_m} + \delta_2 \qquad \text{where } \delta_2 << \omega/V_m$$

and keeping second order terms in δ we find:

$$\delta_2(\delta_2 - \frac{V}{c}\delta_1) = -\frac{\mu c^3}{4d^2(V-c)^3}$$

If we now let $\delta_1 \longrightarrow 0$ and recall that $v-c \cong c$, we see that:

1. The coupling is proportional to ratio of the densities of the two media μ, which is a small quantity, and

2. δ_2 is imaginary

$$\delta_2 = \pm j\left[\frac{\mu c^3}{4d^2(V-c)^3}\right]^{1/2}$$

only if $V > c$

In the system under consideration

$$\frac{\omega}{V_{m+}} = 1.00 \text{ ft}^{-1}$$

and

$$\delta_2 = 0.13 \text{ ft}^{-1}$$

Therefore, $c_{22} = jk_2$ and equations 4.11 reduce to:

$$\frac{\partial a_2}{\partial x} = - jk_2 a_2 + c_{23} a_1^* \tag{4.12a}$$

$$\frac{\partial a_1^*}{\partial x} = c_{32} a_2 + jk_1 a_1^* \tag{4.12b}$$

These equations can be solved by assuming solutions of the form $e^{\gamma x}$ and setting the determinant of the coefficients equal to zero.

$$\gamma_{1,2} = \pm \left[\left(|c_{32}|^2 - \frac{(k_1 - k_2)^2}{2} \right) \right]^{\frac{1}{2}} j \left(\frac{k_1 + k_2}{2} \right) \tag{4.13}$$

where $k_1 = \frac{\omega}{V-c}$ and $k_2 = \frac{\omega}{V_m}$, the undisturbed elemental wave numbers.

Since they are approximately equal:

$$\gamma_{1,2} = \alpha - jk_2 \quad \text{where} \quad \alpha = (\delta_2) = \frac{\mu c^3}{4d^2 (V-c)^2}$$

and the coupled modes can be represented as:

$$a_1^*(x) = A_1 e^{\delta_1 x} + A_2 e^{\delta_2 x}$$

$$a_2(x) = \left| \frac{1}{c_{32}} \right| \left((\delta_1 + jk_1) A_1 e^{\delta_1 x} + (\delta_2 + jk_1) A_2 e^{\delta_2 x} \right)$$

Therefore, $c_{32}^{(2)} = 1\kappa_2^2$ and equations 4.11 reduce to:

$$\frac{dc_2^{(2)}}{dx} = -\frac{1}{2}\kappa_2^2 a_2^{(2)} + c_{32}^{(2)} e^{i\Delta\beta x} \qquad (4.12a)$$

$$\frac{dc_3^{(2)}}{dx} = -c_{32}^{(2)} e^{-i\Delta\beta x} + \frac{1}{2}\kappa_2^2 a_2^{(2)} \qquad (4.12b)$$

These equations can be solved by assuming solutions of the form

$a_2^{(2)}$ and substituting the derivative of the coefficients equal to

zero

$$\gamma = \pm\left[\frac{1}{4}(\Delta\beta)^2 - \frac{\kappa_2^4}{(\Delta\beta - \beta_{32}^{(2)})^2}\right]^{\frac{1}{2}} + \left(\frac{\Delta\beta}{2} - \beta_{32}^{(2)}\right) \qquad (4.13)$$

where $\beta_{32}^{(2)} = \frac{\beta_3^{(2)} - \beta_2^{(2)}}{2}$ and $\Delta\beta = \frac{\beta_3^{(2)} - \beta_2^{(2)}}{2}$, the unperturbed eigenvalue

spacing.

Since they are approximately equal,

$$\gamma = \frac{1}{2}\kappa_2^2 \quad \text{where} \quad \beta_{32}^{(2)} = \frac{\omega_{c_3}(1+\alpha_1)_2}{Hc_2}$$

and the general matrix can be represented as:

$$\varepsilon_+^{(2)}(x) = \psi_+^{(2)} e_2 \xi_2 + \psi_+^{(2)} e_3 \xi_3$$

$$\varepsilon_-^{(2)}(x) = \left[\frac{c_{32}}{\gamma}\right] \left(\xi_2^{(2)} \psi_+^{(2)} e_2 \xi_2 + \xi_3^{(2)} \psi_+^{(2)} e_3 \xi_3 \right)$$

4.7 Summary

The eigenvalue problem has been stated, but was not solved because the method of coupling of modes gave better insight into this problem. We applied coupling of modes to a simplified problem and determined the perturbation of the wave nembers based on the effect of the coupling. In order to properly apply the boundary conditions in this case, the approach of coupling of modes could be followed. But it is not clear that the neglected waves can in fact, be neglected. Since at long wave lengths, the boundaries will have a definite effect on both modes, this must be accounted for in the theory. The results of Chapter III and IV do provide a basis for experimentation. If we assume that the wave numbers are only slightly perturbed; we can compute k_r from the physical length, k_i from equation 4.13 and ω_r and ω_i from the dispersion relationship. Although these values are not exactly self-consistent, they should be approximately so. Experimental techniques to achieve these values are described in the next section.

4. Discussion

The eigenvalue problem has been stated, but was not solved because the method of coupling of modes gave better insight into this problem. The coupling of modes to a simplified problem and determined the characteristics of the wave number based on the effect of the coupling. In order to properly apply the boundary conditions in this case, the spectrum of solutions of modes could be defined. But it is not clear that the solution waves can in fact, be orthogonal. Since at this boundaries will have a definite effect on both modes, this must be accounted for in the theory. The results of Chapter III and IV do provide a basis for understanding. If we assume that the wave numbers are still slightly perturbed, we can consider it from the physical terms, γ_1. Then consider $\gamma_1 i$ and γ_2 and γ_3 from the dispersive relationship. Although these values are not exactly self-consistent, they should be approximately to experimental results to achieve these values are important in the next section.

CHAPTER V

EXPERIMENTAL PROCEDURE

In order to experimentally test the predictions of the theory developed, it is necessary first to insure that the "ingredients" required are available. The initial attempt at assembling a device and making test runs was not successful. The step by step approach will be described in detail. This consisted of several separate experiments. The first was testing of the plate structure to be used. The second was investigation of a means of attaining supersonic flow. Next, the two were combined to test the flutter theory.

5.1 Testing of the Plate

There were two experiments performed on the plate along. The first was an investigation of various types of material and various end conditions The test set up is shown schematically in Figure 5.1 Due to the magnetic coupling between the two circuits, the results of this test were qualitative rather than quantitative. The results are shown in table 5.1. The end conditions have a noticeable effect on the response. The damping or losses in any condition but tightly clamped appeared to nulify any response.

The frequency did increase with tension which indicates that the use of membrane analysis in the coupling of modes section is not totally in error. The frequency of response was reasonably

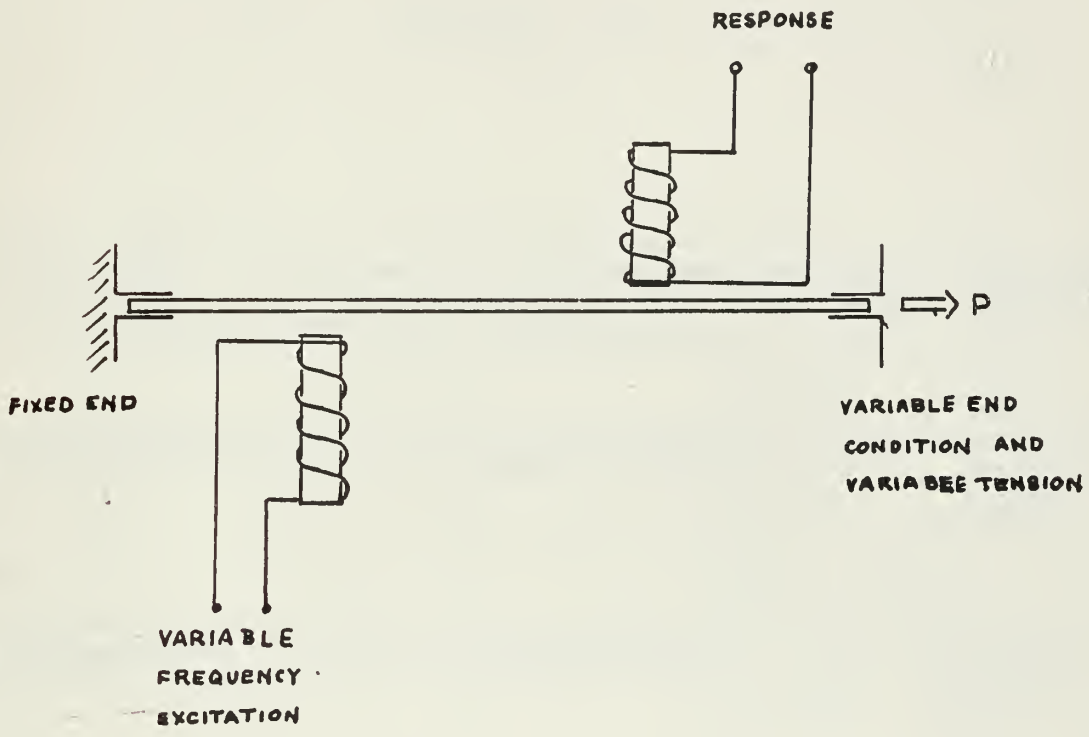
EXPERIMENTAL PROCEDURE

In order to experimentally test the predictions of the theory developed, it is necessary first to obtain two "interfaces" required are available. The initial attempt at assembling a device and making such was not successful. The step by step approach will be described in detail. This consisted of several separate experiments. The first was testing of the glass structure to be used. The second was investigation of a means of obtaining electronic flow. Next, the two were combined to test the Dexter theory.

4.1 Testing of the Glass

There were two experiments performed on the glass alone. The first was an investigation of various types of material and thickness and conditions. Therefore was to be shown schematically in Figure 3.1. Due to the magnetic coupling between the two electrodes, the results of this test were qualitative rather than quantitative. The results are shown in Table 3.1. The end conditions have a noticeable effect on the response. The thinner or leaves [illegible] somewhat apparent to unify any response

The frequency did increase with power, which indicates that the use of conductive samples in the coupling of power together is not totally in error. The frequency of response was consistently



TOP VIEW OF TEST SECTION

FIGURE 5.1

TABLE 5.1

SAMPLE	DIMENSIONS	END CONDITIONS	FREQUENCY	RESPONSE
1 COLD ROLLED IRON	6×1×.030 (inches)	CLAMPED-CLAMPED	51 (CPS) 103 300	SMALL SMALL SMALL
2 HEAT TREATED STEEL	8×1×.015	CLAMPED-CLAMPED	63 112	LARGE MEDIUM
3 SPRING STEEL	6×1×.015	CLAMPED-CLAMPED	70 140	MEDIUM SMALL
SAMPLE 2		CLAMPED-PINNED NO TENSION	—	NONE
SAMLE 2		CLAMPED-PINNED TENSION	75 148	LARGE MEDIUM

close to that predicted. (See example 5.1)

EXAMPLE 5.1 $$\omega_m^2 = \pi^2 \left(\frac{m^2}{\ell^2} + \frac{n^2}{w^2}\right) \frac{P}{\mu}$$

where

D = EI, the rigidity

μ = mass per unit area

M = longitudinal harmonic mode

ℓ = length

$\omega_2 = 650 \frac{1}{\text{sec}}$

$f_2 = 104$ cps

The second experiment on the plate above was to determine if in fact the second harmonic frequency could be excited when the media had the perdominate properties of a membrane. Equipment similar to that shown in Figure 5.1, was used for this part of the experimentation To avoid the magnetic coupling, one of the electromagnets was replaced by a speaker for mechanical excitation with weak coupling. The plate thickness was reduced to .001 inch, to more closely approximate a membrane. The equipment is shown pictorially in Figure 5.

In this configuration the plate frequencies were not detectable The membrane frequencies gave excellent response and are tabulated in Table 5.3 The variation between predicted and

close to that predicted. (See Example 5.1)

EXAMPLE 5.1 $\omega_m^2 = \epsilon_2 \frac{k_2^2}{\ell^2} = \frac{D_2}{\mu_2} \frac{k_2^4}{\ell^4}$

where

D = EI, the rigidity

μ = mass per unit area

k = longitudinal harmonic mode

ℓ = length

$\omega_m = 650 \frac{1}{sec}$

$\epsilon_2 = 104$ cps

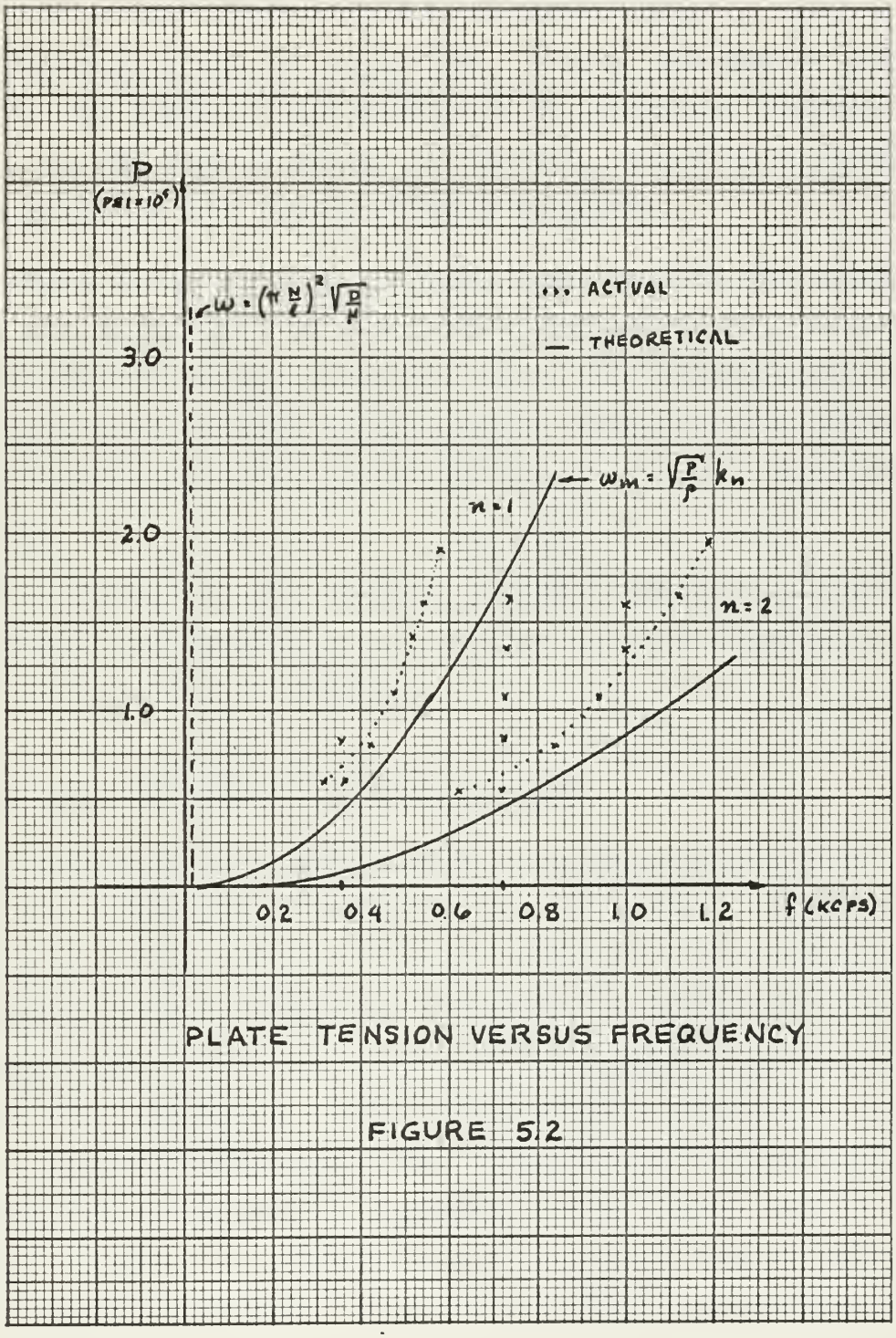


PLATE TENSION VERSUS FREQUENCY

FIGURE 5.2

actual frequencies can be explained. First, the actual parameters for the material are difficult to determine exactly. Secondly, at the higher values of tension, there is a possibility that the grips or end clamps slipped slightly.

The main conclusion to be drawn here is that the second harmonic can definitely be observed. The quality of the response decrease with increasing frequency since the higher modes are more difficult to exert. The frequency expected when operating the wind tunnel could be reasonably predicted from this type of experiment

The responses at 355 and 720 cps are probably due to weak coupling with the speaker or other portion of the apparatus, since they are independent of tension.

5.2 Experiment to Obtain Supersonic Flow.

The physics of supersonic flow is a very complicated subject and is extensively covered in the literature, notably in reference 15. Only those principles which are germane to this experiment will be touched on. In general, supersonic flow can not be attained directly from an air compressor. It is necessary to modify the compressor output by passing it through a set of nozzles. For this experiment the nozzles were of the convergent-divergent type. The design procedure is to attain Mach 1 flow at the throat of the test section and then expand to the velocity desired. The calculations for this are shown in example 5.2,

based on the set of nozzles shown in Figure 5.4.

EXAMPLE 5.2 (Reference 15, Table BIII)

Inlet Conditions:

$$P_o = 125 \text{ psia} \qquad T_o = 70^\circ F$$

a. Static Conditions:

$$\rho_o = \frac{P_o}{RT_o} = 0.605 \ \frac{m}{ft^3}$$

$$c_o = 49.1 \ T_o = 1158 \ \frac{ft.}{sec.}$$

b. Throat Conditions:

$$P^*/P_o = 0.528$$

$$T^*/T_o = 0.823$$

$$\rho^*/\rho_o = 0.634$$

$$c^*/c_o = 0.913$$

c Test section conditions for Mach No. = 2.0

$M^* = 1.633$ $\qquad A/A^* = 1.687$ $\qquad A_{test} = 6.0 cm^2$

$P_1/P_o = 0.1278$ $\qquad V = 1733$ ft/sec $\qquad A_{throat} = 3.57 cm^2$

$\rho_1/\rho_o = 0.2300$ $\qquad P_1 = 16.1$ psia

$T_1/T_o = 0\ 555$

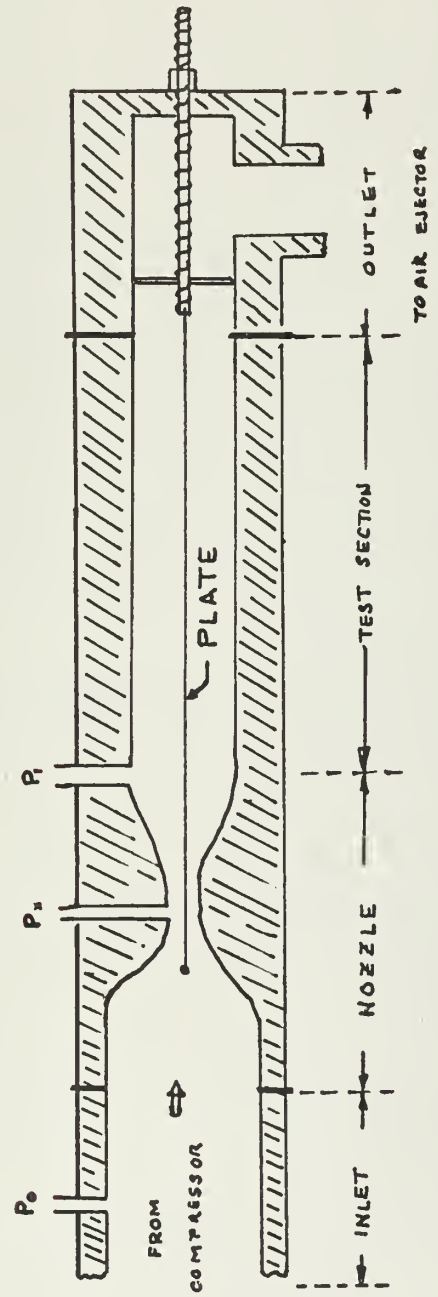
d. Mass Flow

$A^* \ ^4 \ x60 = 245$ cuft/min.

Secondly, the flow in the test section will be greatly effected by the build up of the boundary layer. This has the effect of a convergent nozzle, which tends to reduce to flow velocity toward Mach 1. In the case of Mach 2 flow, the velocity will be reduced to Mach 1 in a length of 31 diameters. In this experiment, the diameter or depth of the channel will be taken as one centimeter to allow sufficient magnetic coupling. Therefore, the length must be less than 31 centimeters; an initial length of 20 centimeters was used. (From this length, the thickness of the plate for a desired natural frequency can be computed as in example 5.1). Based on these calculations, the experiment shown schematically in Figure 5.3 and 5.1 was constructed.

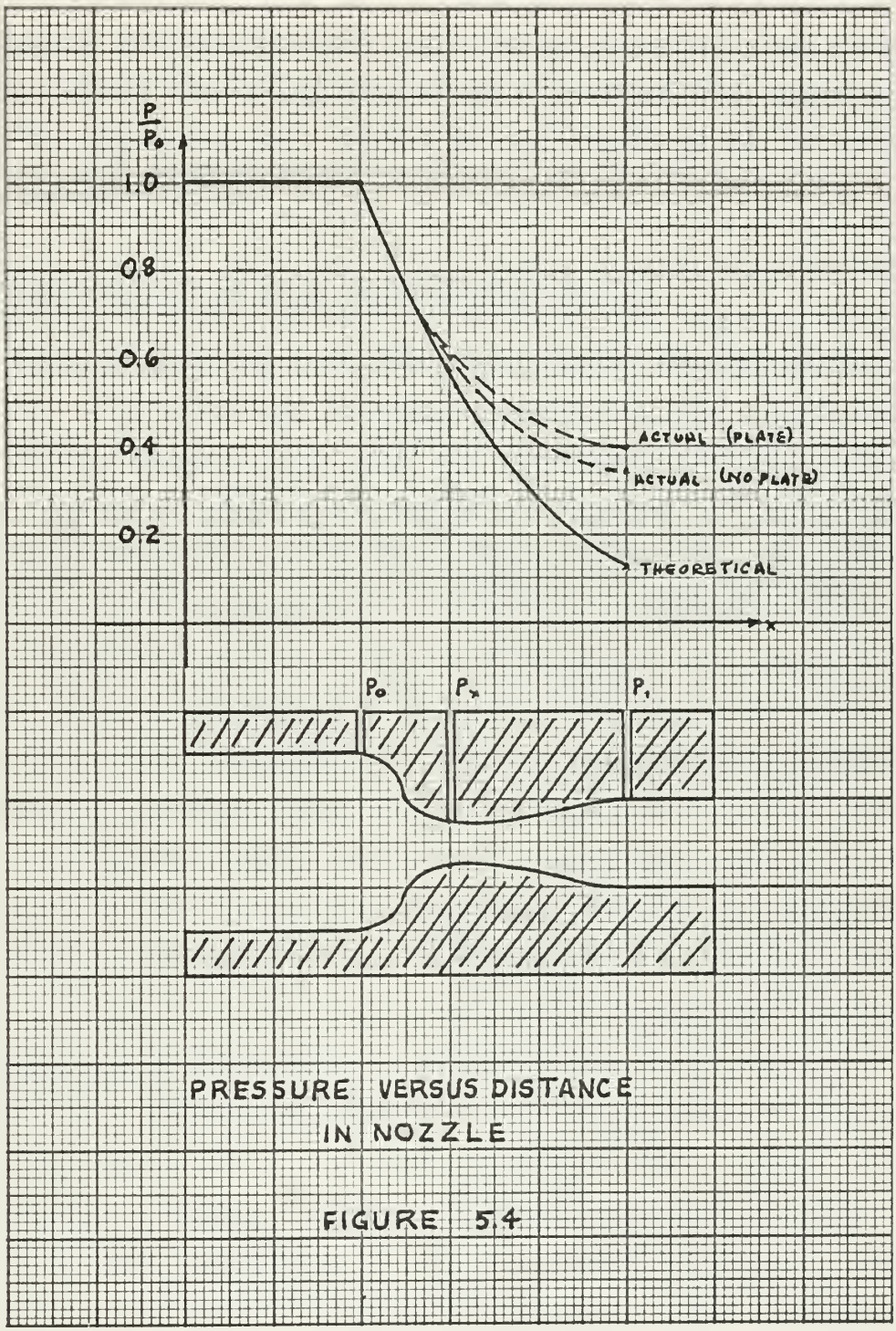
The pressure readings were taken with both the plate in position and removed. The readings were taken with a guage and compared with a monometer for accuracy. The readings obtained are shown in Figure 5.4.

These results are at a variance with those predicted in example 5.2, but they do indicate supersonic flow. The variance is due to several factors. First, the ideal area ratios are significantly diminished by the boundary layer to an "effective" area ratio. Secondly, the capacity of the air ejectors was insufficient for running the experiment as desired. Also, the pressure measurements were made at the wall of the channel, probably in the boundary layer, and not in the center of the flow. The latter problem is difficult to circumvent, and in previous experiments (reference 15, page 135) good agreement between actual



NOZZLE TEST SECTION

FIGURE 5.3



PRESSURE VERSUS DISTANCE IN NOZZLE

FIGURE 5.4

wall pressure and theoretical mid-stream pressures has been attained. Using the lower pressures recorded, the speed of the flow can be approximated. Based on the readings at 35 psia inlet, the flow is Mach 1.47.

5.3 An Experiment to Test the Flutter Theory

We have determined that the ingredients of the theory are attainable. The flow can be made supersonic, the plate can be excited in the second mode, and the frequency of this mode varies with the tension. Thus the results of Chapter IV indicating that the perturbation on the wave number k can be estimated from the coupling seem reasonable. The rigidity term Dk^2 can be neglected compared to the membrane term M_m^2 at the frequencies in question. This is especially true if the very thin (.001") plate is used. The first experiments with the combined ingredients used a .015 inch plate. The equipment consisted of the nozzle, a plate, a magnetic detection device and an oscilloscope. The presentation on the scope was confused, showing standing waves and traveling waves and these responses were at several frequencies. These were probably excited by the boundary layer. The tension was varied and the frequencies of the response increased with increasing tension. By varying the pressure the response would set in at a certain pressure for a given tension - quantitatively a higher tension required a higher pressure. At a given pressure, providing it was above the threshold, the varying of the tension was limited mechanically. That is, the tension

could be increased until the plate failed in tension, and if the tension was decreased the plate would fail by excessive displacement. The latter occured at low values of tension. It was felt that nothing of substance to support the theory could be gained from further experimentation with this set up. "Flutter" did occur and this did provide a variable coupling between the two magnetic circuits and there was an output signal.

However, this flutter could not be attributed exclusively to the mechanism proposed in the theory. It was probably caused by excitation from the boundary layer. There was a component in the vicinity of the frequency expected as computed below:

(1) length of membrane: $k = \frac{2\pi}{L} = .628 \text{ in}^{-1}$

(2) from dispersion equation: $f = \frac{(M_a - 1)kv_o}{2\pi} = 1080 \text{ cps}$

$$\omega_i = 495 \text{ sec}^{-1}$$

(3) from coupling of modes: $\alpha = \frac{\mu c^3}{4d^2(v-c)^2} = .25 \text{ in}^{-1}$

In order to prevent the end connections from interferring with the flow they were removed from the test section as shown in the figure. Therefore, the flow is in transition over a short

section at both ends of the plate. This could also provide an excitation which was not predicted and would be difficult to analyze.

Therefore, another experiment was designed. Due to a time problem, it may not be completed before the due date of this thesis. The changes include using a .001 inch plate to more closely approximate membrane performance, refined end connectors to insure a good response and no flow disturbance by the tension mechanism. The results of this phase of the experimentation will be reported in Appendix B.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The analysis of the model does predict the desired phenomenon. The interaction between the flow and the elastic media at long wavelengths will theoretically produce growing oscillations. The effects of the longitudinal boundary conditions can be approached by the coupling of modes. In the simplified problem considered in Chapter IV, the effect of the coupling on the wave number was shown. This approach could be expanded to show the influence of the boundaries on the wave numbers.

The experimental portion shows that the separate systems can be achieved. The plate can be excited in the second mode. The natural frequencies of the plate increase with tension and the tension required by the theory is physically realizable. A supersonic flow over the plate can also be attained in the laboratory. With the equipment used, the maximum velocity was 1.47 times the speed of sound. The major source of difficulty encountered in combining the systems was the excitation provided by the boundary layer. This was not included in the model or in the analysis. The excitation gave forced responses, mainly in the first mode, although there was a second mode in the response.

There are several other considerations which have not been covered. With respect to the elastic media, there will be eddy currents induced when it oscillates in the magnetic field.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The analysis of the model does contain the desired phenomena. The interactions between the flow and the elastic media at long wavelengths will theoretically produce growing oscillations. The effects of the longitudinal boundary conditions can be approximated by the coupling at modes. In the stability problem considered in Chapter IV, the effect of the coupling on the wave number was shown. This approach could be modified to show the influence of the boundaries on the wave numbers.

The experimental portion shows that the predicted effects can be achieved. The plots can be examined in two manners. The natural frequencies of the plate interact with transition and the results depicted by the theory is physically realizable. A experimenter like myself can also be attached to the laboratory. With the equipment used, the maximum velocity was 1.47 times the speed of sound. The major sources of difficulty encountered in examining the systems was the reflection provided by the boundary layers. This was never considered in the model or in the analysis. The oscillation gain depend phenomena, mainly in the first mode, although there was a second mode in the response.

There are several other considerations which have not been covered. With regards to the elastic media, there will be only current induced when it oscillates in the magnetic field.

These represent a loss in the system which could be significant. The Curie temperature will impose a thermal limitation, restricting efficient operation of the device to relatively low temperatures. From the stand point of strength, it is fortunate that ferrous materials, which are required magnetically, also show good fatigue properties.

The most severe limitation on the device efficiency will be the losses incurred by the fluid in transiting the nozzle and interaction area. It was noted in the experimental section that the flow would be supersonic only for a short distance. For instance, the Mach 1.47 flow in the experiment would be "choked" to Mach 1.0 at the end of the test section (six inches), due to boundary layer build up. At the same time the isentropic stagnation pressure would be reduced to less than one half its initial values. This represents a considerable loss in efficiency. A more refined design would improve on this situation, but there will be a definte limiting efficiency attainable.

The subject of supersonic flow in short ducts is somewhat complex and experimental data has not been consistant.[15] In general, the boundary layer thickness increases with length. But in some cases, negative friction coefficients have been attained, indicating a decreasing boundary layer. The effects of the oscillating plate on the boundary layer have also been studied but there appears to be no significant change in boundary layer thickness.[16]

The device itself may not be the most efficient way of obtaining energy from the flow. However, it is possible that to employ it in tandem with a more conventional energy converter would improve overall efficiency. The important conclusion here is that a mechanism is available for transfer of energy between the flow and the media. The mechanism itself is not a "lossy" one - the inefficiencies are incurred in the scheme for using the mechanism attempted here. In other words, this paper has been limited in scope to a particular device, but further research would lead to other methods of using this concept for energy transfer.

6.2 Recommendations

The study of the longitudinal boundary conditions in this scheme could be expanded using the coupling of modes approach. The coupling coefficients considered here pertained only to the coupling between two waves, one in each system. The coupling between the other two waves, both with the other wave in its own system and the other wave in the other system were neglected. The latter still appears reasonable. But the boundary conditions will provide coupling between the two waves in one system. We neglected this here because at the wavelengths in question, the rigidity term could be neglected and the plate considered as a membrane. In the general case this would not be so, and the area of longitudinal boundary conditions in coupled systems has many possibilities for study.

The combined experiment showed the overstanding effect of the boundary layer, essentially masking the desired response if present. The boundary layer could be reduced either by thermal methods or by expanding the flow in the depth dimension. If this were done, the correlation of experiment and theory might be attained and the mechanism studied.

Ketterer[9] used a fluid stream and a spring coupled electrically to study this mechanism. Fraize[17] studied a device in which a fin, situated in a wind tunnel (sub sonic), was excited mechanically to produce traveling waves. Significant reductions in drag were recorded. Replacing the mechanical excitation by magnetic traction along the length, using the coupling mechanism investigated here, provides the possibility for pumping of or propulsion through a fluid surrounding the fin

This would be the inverse of the problem studied here, but would not have the boundary layer problem inherent in super sonic flow.

Appendix A

A COMPARISON WITH FLUTTER THEORY

Flutter is defined as the self-excited oscillations of a airfoil in supersonic flow.[18] This definition leaves some doubt as to the physics involved, but since it is restricted to supersonic flow it seems conceivable that the mechanism involved is similar to the feedback situation described in Chapter III.

The problem of flutter is stated as a partial differential equation, where structural, inertial, and also dynamic operators are combined to determine the equation of motion for the panel. Two approaches are followed from here depending on the geometry involved. One is to apply the boundary conditions to the plate only. Since these are two at each end, a "fourth order" eigenvalue problem results.

The second approach is to assume traveling wave solutions, $e^{jkx-\omega t}$, and assuming real values for k, compute complex values of ω. Dugundji[19] has used both approaches and compared the results. He then recommends which approach to use based on the geometry of the problem considered.

In terms of our parameters, the equation of motion is:

Appendix A

$$D\frac{\partial^4\xi}{\partial x^4} - P\frac{\partial^2\xi}{\partial x^2} + 2\rho\Delta\frac{\partial^2\xi}{\partial t^2} + G_s\frac{\partial\xi}{\partial t}$$

$$= \frac{\rho c^2 V_o}{(1-M_a')^{1/2}}\left(\frac{\partial\xi}{\partial x} + \left(\frac{1-2M_a^2}{1-M_a^2}\right)\frac{1}{V_o}\frac{\partial\xi}{\partial t}\right)$$

where the term on the right is the "aerodynamic pressure." For small M_a, this reduces to

$$-\rho c^2\left(\frac{\partial\xi}{\partial t} + V_o\frac{\partial\xi}{\partial x}\right)$$

and the equation is the same as equation (2.10) except for the structural damping term G_s. In our case, the aspect ratio is zero and the first approach is recommended. That is: the boundary conditions on the plate are applied and a solution of the form

$$\xi = (\xi_1 e^{k_1 x} + \xi_2 e^{k_2 x} + \xi_3 e^{k_3 x} + \xi_4 e^{k_4 x})\, e^{j\omega t}$$

is assumed. This forms the eigenvalue problem, and the solutions for ω_r, ω_i are shown in reference 18. Many parameters have been introduced and this leads to lengthy computations to compare the

the results of the two theories. For the tension, rigidity, and plate thickness employed in the device, the solutions predict $\omega_i = 100$ and $\omega_r = 1000$. The approximation is necessary because the plotted data must be extropolated to include the values of tension specified. These results show correlation within an order of magnitude of those predicted in Chapter V. ($\omega_r =$, ω_i). Most of flutter investigation has been restricted to determining the onset of flutter and methods for avoiding it's occurance. Again using our parameters in figure 10 of reference 18, flutter is predicted to occur when the tension corresponds to making $v_m + = v\text{-}c$. This is surprisingly consistent with the previous results. The predicted flutter frequency under these conditions is on the order of $20\omega_o$ where ω_o equals the natural plate frequency neglecting tension. The previous results predict $\omega_f = 10\omega_o$.

There is considerable agreement between the two results, in spite of the difference in boundary conditions.

Appendix B

The previously described experiment was constructed. However, the improved end conditions to lessen the flow disturbance were mechanically unweildly. They did not provide the firm connection desired. Therefore, no further results of significance can be reported.

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